BASIC ACOUSTICS

Instructor
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P773 Acoustics
June, 2008

1) THE GAS LAW

2) PLANE WAVES

3) SPHERICAL WAVES

4) PROPAGATION OF SOUND

5) GENERATION OF SOUND

6) DIRECT-RADIATOR LOUDSPEAKERS

University of Waterloo

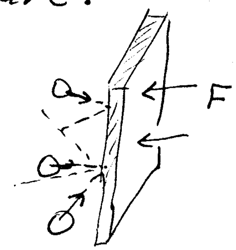
THE GAS LAW: $PV = RT$

ATMOSPHERIC PRESSURE $\sim 10^5\ N/m^2 \sim 10^4\ kg/m^2$

Density $\sim 1.2\ kg/m^3 \rightarrow$ column 8km high

Molecular bombardment of vessel wall produces force representing pressure.

pressure $\propto$ # impacts/sec $\times$ speed



— Density of molecules $\propto \frac{1}{Volume}$

So if speed is constant:

$$P \propto \frac{1}{V}$$

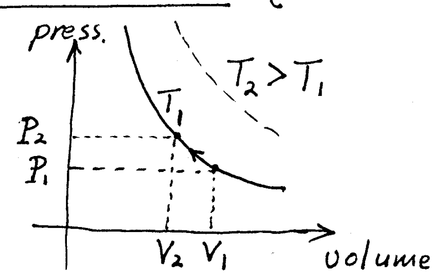
— Increasing speed by factor of 2 doubles effect of impacts, and doubles #/sec.

Thus $P \propto v_{rms}^2$ and both factors give $PV \propto v_{rms}^2 \propto$ energy $\propto T$ (absolute temp.)

$$PV = RT$$

ISOTHERMAL PROCESS (fixed temp.)

$PV =$ constant



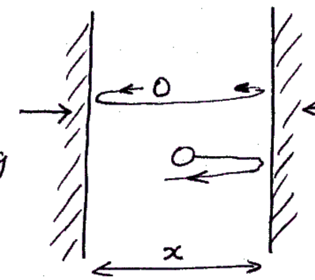
Physical principle

ADIABATIC PROCESSES

Changes in p & V without significant heat flow in or out. Sound fluctuations are adiabatic.

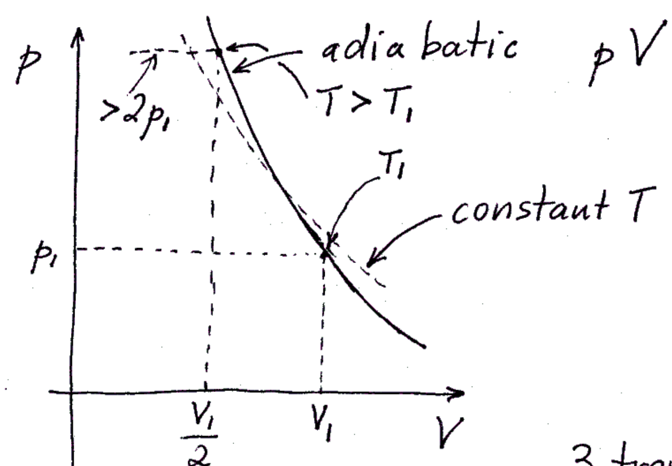
Simplified Example

moving in hard walls cause bouncing ball to increase its speed.



speed $\propto \frac{1}{x}$

In a diatomic gas, molecular energy goes into rotational (and vibrational) modes.



$pV^{\gamma} = \text{constant}$

For a diatomic gas: 3 translational, 2 rotational modes

$(0.8)N_2 + (0.2)\,O_2$

AIR

specific heats $\rightarrow C_V$

$$\gamma = \frac{C_p}{C_V} = \frac{2+5}{5} = 1.40$$

for air

TEMPERATURE OSCILLATIONS

As sound waves come by a particular point, the pressure changes sinusoidally, as does the temperature.

$$\frac{\Delta T}{T} = \left(\frac{\gamma - 1}{\gamma}\right)\frac{\Delta p}{p_0} \simeq 29\% \text{ of } \frac{\Delta p}{p_0}$$

for air.

Denotes an Example

For sound of 120 dB SPL,

$$\frac{\Delta p}{p_0} \sim \frac{1}{5000}$$

So $\frac{\Delta T}{T} \sim \frac{1}{17000}$

T = absolute temp $= 273 + T_{°C} \sim 300°K$

So $\Delta T \sim \frac{1}{60}$ °C

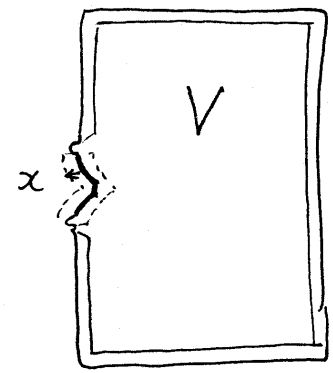
But this temperature fluctuation is important: it changes the speed of sound, and is responsible for the decay of sound waves.

PRESSURE IN SEALED BOX

$$\Delta V = A_0 \, x$$

cone area

displacement of cone



At low frequencies, air in box acts as a spring, and the adiabatic equation of state of the air must be used, which gives:

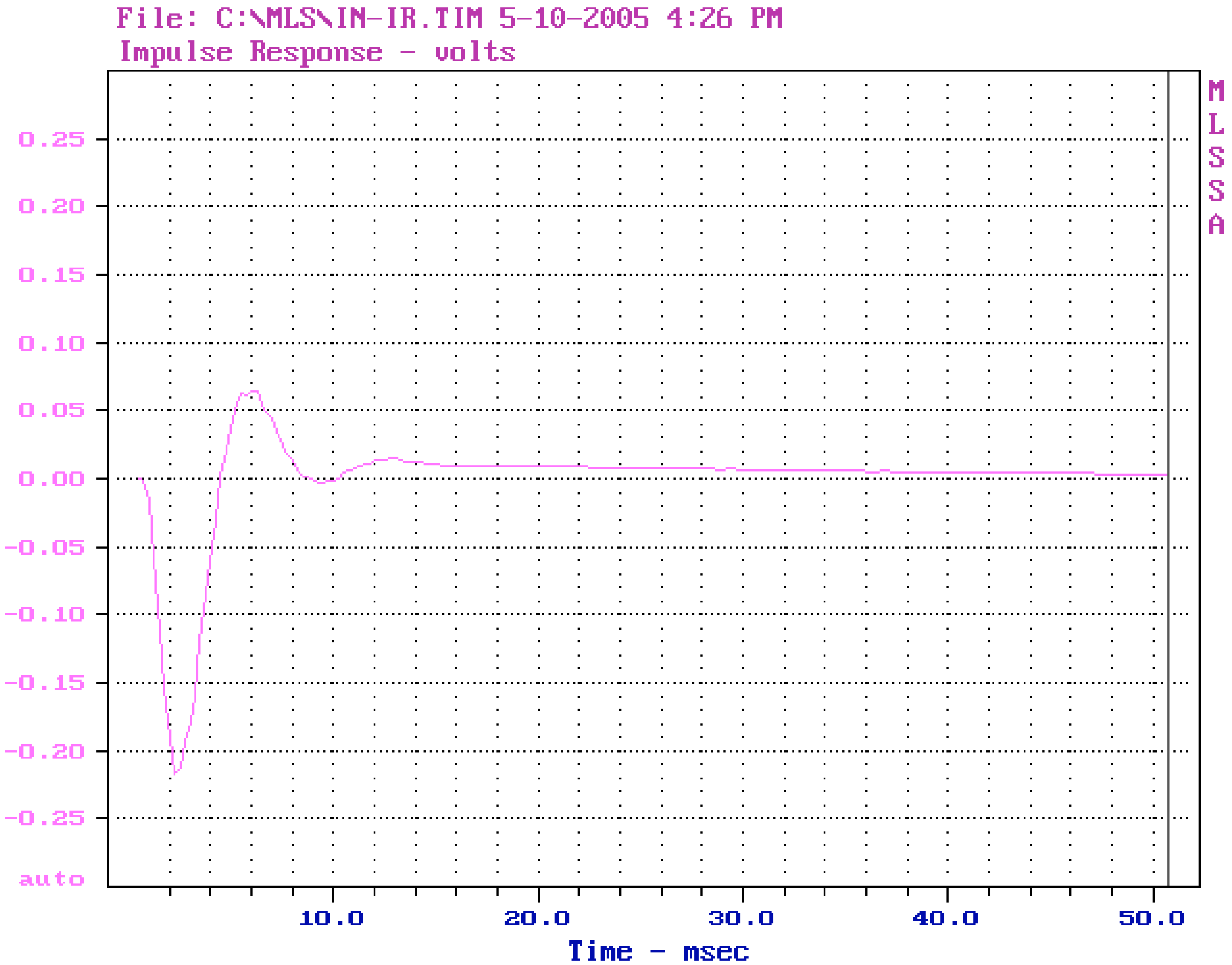
$$\frac{\Delta p}{p_0} = -\gamma \, \frac{\Delta V}{V}$$

Example: 25 liter box (~ 1 cubic foot)
20 cm driver (8 inch)
$x = 1$ mm

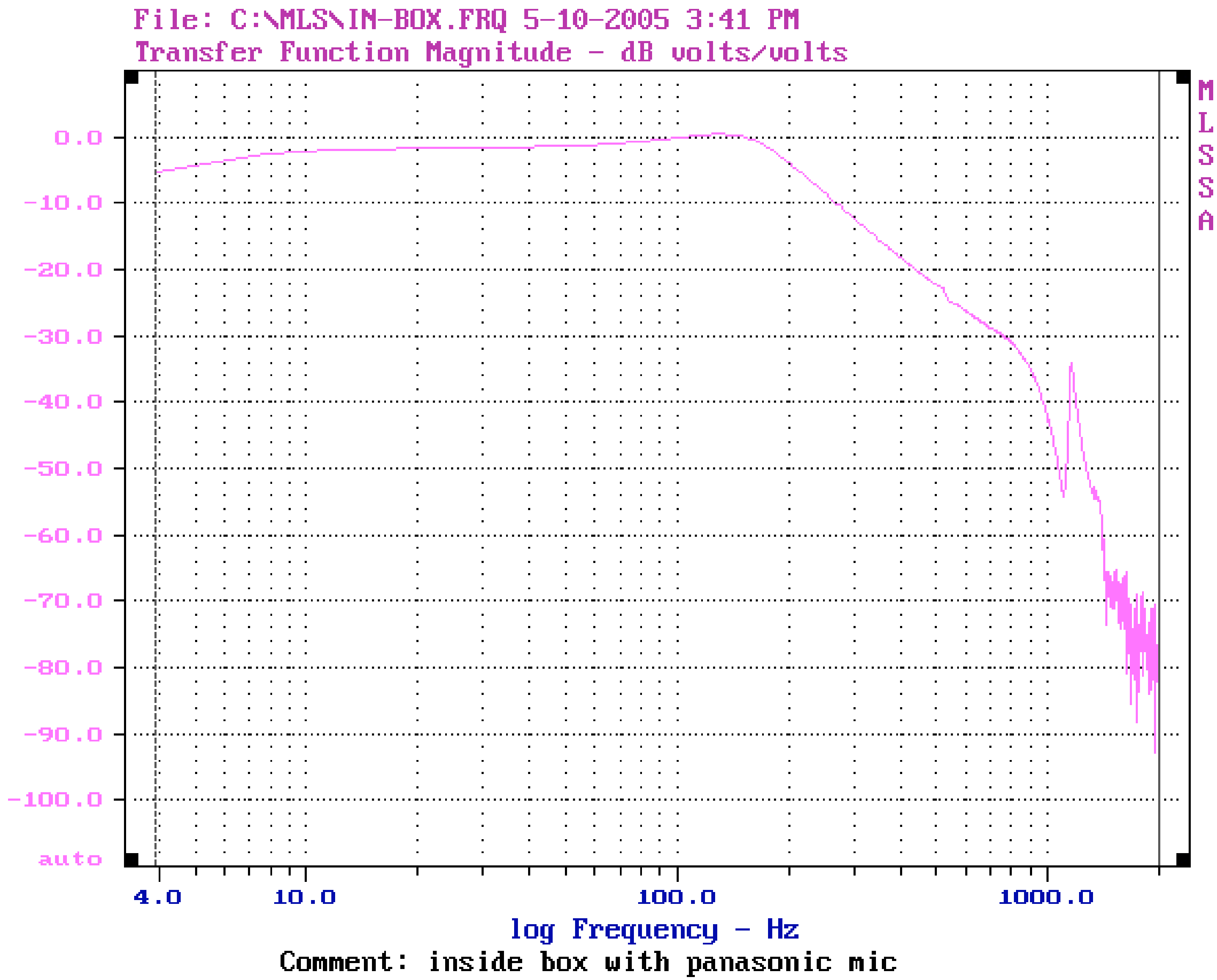
gives $\Delta p = 176$ Pa peak
→ 136 dB (equiv SPL)

at low frequency, inside the box.

Measure pressure inside sealed box

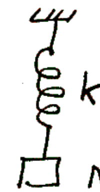


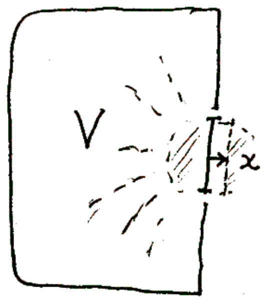
Acoustic impulse response inside the box


File: C:\MLS\IN-BOX.FRQ 5-10-2005 3:41 PM
Transfer Function Magnitude - dB volts/volts
MLSSA
0.0
-10.0
-20.0
-30.0
-40.0
-50.0
-60.0
-70.0
-80.0
-90.0
-100.0
auto
4.0
10.0
100.0
1000.0
log Frequency - Hz
Comment: inside box with panasonic mic

RESONANCE

The cone (and the air very nearby) has mass, while the air in the box acts like a spring.





$$\frac{\Delta p}{P_0} = -\gamma \frac{\Delta V}{V} = -\gamma \frac{A_0 x}{V}$$

compare with spring

$$\Delta F = -k x$$

Example:

25 liter box
20 cm driver
20 g moving mass
ignore driver surround

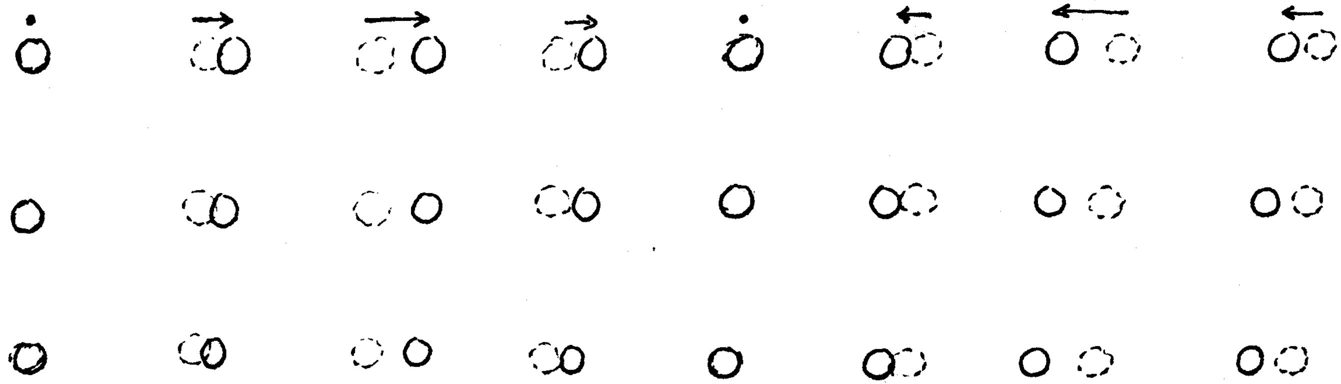
$$f_{res} = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0 A_0^2}{M V}} \sim 84 \text{ Hz}$$

Air spring only

A common mistake is to use too large a driver in a modest box.

PLANE WAVES

PICTURE OF DISPLACEMENT
VELOCITY
PRESSURE



⟹ DIRECTION OF WAVE MOTION ⟹

|← wave length →|

← ← • → ⟶ → • ←

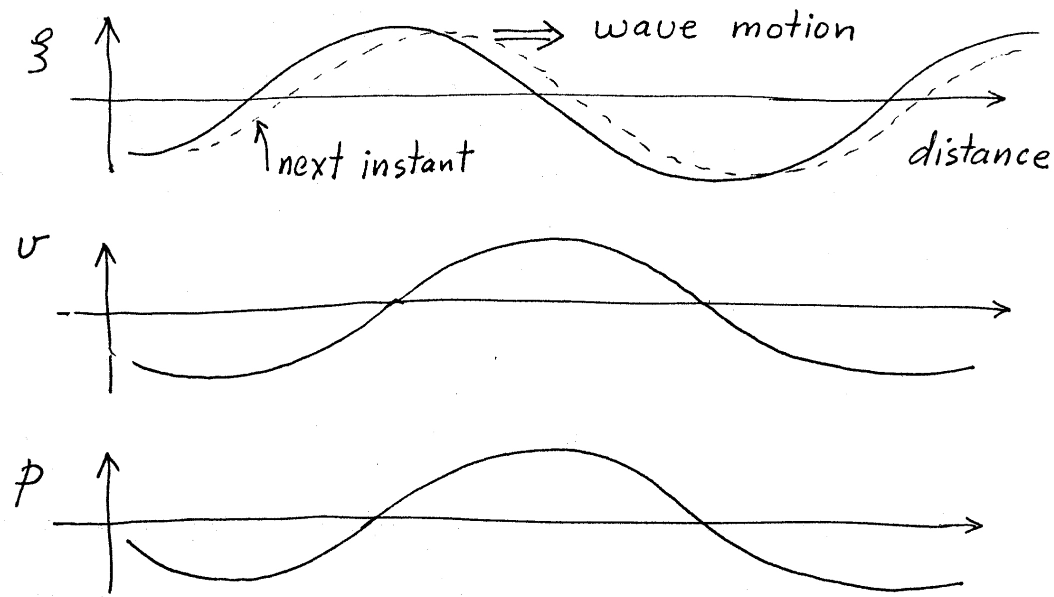
VELOCITY OF MOLECULES

−high −med Ø +med +high +med Ø −med

PRESSURE (and TEMPERATURE)

[THE CHANGES ACTUALLY]

RELATIONSHIPS OF ξ, v, p For Plane Waves



DISPLACEMENT ξ IS 90° LAGGING
THE VELOCITY v

PRESSURE p and VELOCITY v
ARE IN PHASE

WE NORMALLY CONSIDER THE PRESSURE p TO BE JUST THE SMALL FLUCTUATION WE CALL SOUND, OFTEN WRITTEN Δp.

IF $\frac{\Delta p}{p_o} \sim 10^{-5}$ SPL $\sim$ 91 dB

SPEED OF SOUND : TEMP. DEP.

$$\text{speed} \propto \sqrt{\frac{\text{ELASTIC FACTOR}}{\text{INERTIAL FACTOR}}}$$

$$c = \sqrt{\frac{\gamma p_0}{\rho_0}}$$

effect of adiabatic process

atmospheric pressure

atmospheric density

Now $\rho \propto \frac{1}{V}$

Thus $c \propto \sqrt{\gamma p V}$

$\propto \sqrt{\gamma T}$

T	c [m/s]
40°C	355.5
20°C	344
−40°C	306.8

CONCLUSION: The speed of sound does not depend on air pressure or density, but only on $\sqrt{\text{absolute temperature}}$

Now KE of molecules $= \frac{1}{2} M v_{rms}^2 = \frac{3}{2} kT$

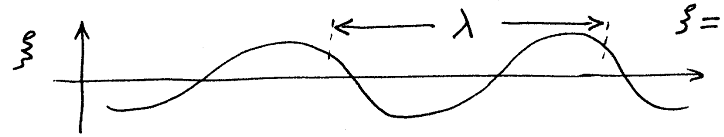
Theory tells us $c = \sqrt{\frac{\gamma}{3}}\, v_{rms} = 68\%\ v_{rms}$

See D. Bohn
JAES 36 April 1988
"ENVIRONMENTAL EFFECTS ON THE SPEED OF SOUND

More later…

TRAVELLING WAVES

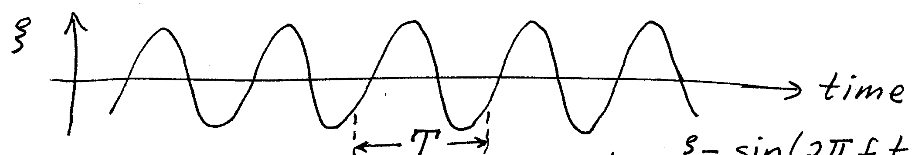
SNAPSHOT OF WAVE AT PARTICULAR TIME:



$\xi = \sin\left(\frac{2\pi x}{\lambda}\right)$

distance

VARIATION WITH TIME AT PARTICULAR PLACE:



$T = \frac{1}{f}$ $\qquad \xi = \sin(2\pi f t)$

EQUATION:

$$\xi = \xi_0 \sin\left(\underbrace{2\pi f}_{\omega} t \pm \underbrace{\frac{2\pi x}{\lambda}}_{k}\right)$$

$$= \xi_0 \sin(\omega t \pm k x)$$

$\boxed{\text{speed } c = f\lambda = \frac{\omega}{k}}$

ξ_0 = displacement amplitude

$+ \Rightarrow$ left moving wave

$- \Rightarrow$ right moving wave

ω = angular frequency

$k = \frac{2\pi}{\lambda}$ = wave number

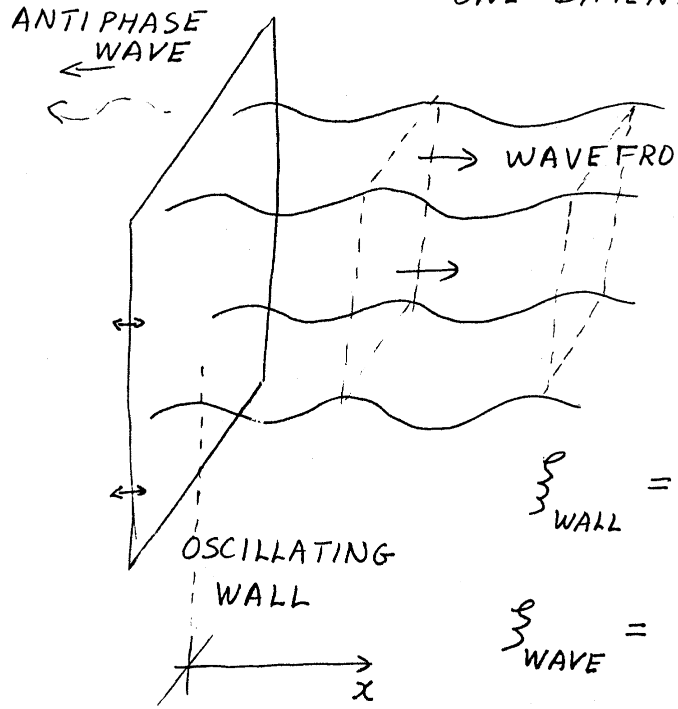
Mathematical principle

$$v = \lim_{\Delta t \to 0} \frac{\Delta \xi}{\Delta t} = \omega \xi_0 \cos(\omega t \pm k x)$$

also travelling wave, but leading in time phase by 90°.

GENERATING A PLANE WAVE

"ONE DIMENSIONAL" SOUND SOURCE



AMPLITUDE OF VIBRATION

Along the wave direction, longitudinal

$$\xi_{WALL} = \xi_0 \sin(2\pi f t)$$

$$\xi_{WAVE} = \xi_0 \sin\left(2\pi f t - \frac{2\pi x}{\lambda}\right)$$

$$\xi_{WAVE} = \xi_{WALL} \quad \text{at} \quad x = 0$$

FROM $v = \left.\frac{\Delta \xi}{\Delta t}\right|_{x=const}$ and $p = -\gamma p_0 \left.\frac{\Delta \xi}{\Delta x}\right|_{t=const}$

WE CAN SHOW $p = \rho_0 c\, v$ for a plane wave

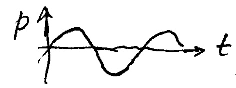
SO p and v ARE IN PHASE

$\frac{p}{v}$ = ACOUSTIC IMPEDANCE $= \rho_0 c \sim 406 \frac{N\text{-}s}{m^3}$ @ 20°C

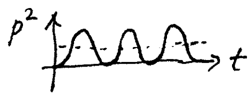
(PURELY RESISTIVE) "RAYLS"

INTENSITY, IMPEDANCE

$$\text{INTENSITY} = \frac{\text{ENERGY FLOW/SEC}}{\text{UNIT AREA}} = \frac{\text{POWER}}{\text{UNIT AREA}}$$



$$= \frac{F\,v}{A} = \frac{A\,p\,v}{A} = p\,v$$



$$= \rho_0 c\, v^2 = \frac{p^2}{\rho_0 c} \quad \text{(INSTANTANEOUS)}$$

AVERAGE INTENSITY (for sinusoid) $= \dfrac{p_{peak}^2}{2\rho_0 c} = \dfrac{p_{rms}^2}{\rho_0 c}$

EXAMPLE

SUPPOSE A WALL HAS 1 mm peak MOTION AT 100 Hz.

$v = 2\pi f \xi_0 \rightarrow 0.628$ m/s peak

$p = \rho_0 c\, v \simeq (1.2)(344)(0.628)$

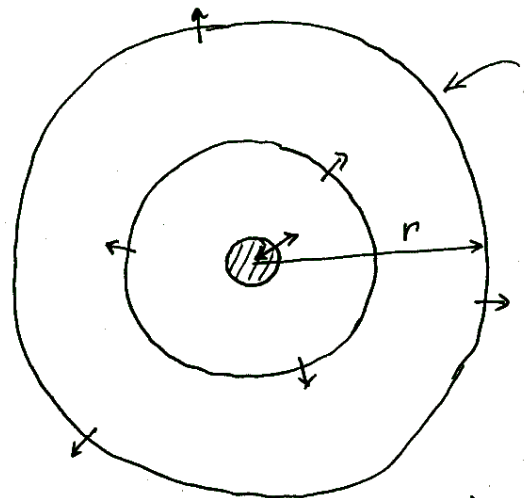
$\simeq 259$ Pa peak

$\rightarrow 139$ dB SPL

1 mm peak motion at 1kHz would produce 159 dB SPL, but would require 100 times the force on the wall.

SPHERICAL WAVES

Consider a radially oscillating sphere.



expanding spherical waves

The waves spread out at the speed of sound, and the pressure shape does not change, except that the wave weakens as the distance from the source increases.
(from wave eqn in 3 Dimensions)

– radius of wavefront = $c\,t$

The pressure can be described by:

any function $\rightarrow$ $$p = K\frac{f\left(t - \frac{r}{c}\right)}{r}$$

[A pulse would remain a pulse.]

At $r=0$, p would be infinite, but the actual source surface would be at a finite radius.

Motivate this with energy conservation

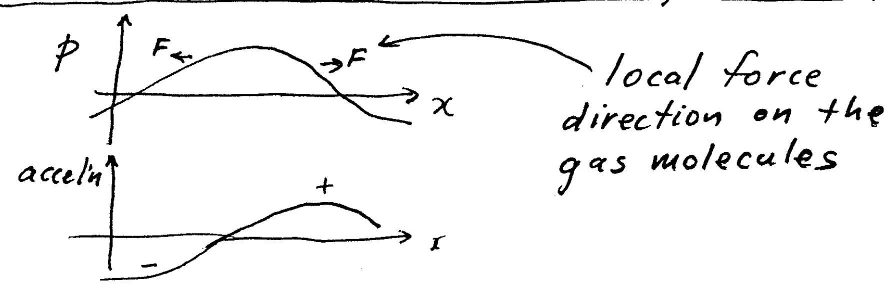
COMPACT SPHERICAL SOURCES

Accepting that $p = \frac{K}{r} f(t - r/c)$, let us try to determine what K and the function $f(\ )$ is by using Newton's Law.

2nd Law: (mass) x (acceleration) = Force

In a gas this becomes:

$$\boxed{(\text{density}) \times (\text{acceleration}) = -\begin{pmatrix}\text{rate of change}\\ \text{of pressure}\end{pmatrix}}$$



For our spherical wave then:

$$\boxed{\rho_0 \begin{pmatrix}\text{radial accl'n}\\ \text{of air molecules}\end{pmatrix} = -\frac{d}{dr}\left\{ \frac{K}{r} f(t - r/c)\right\}}$$

The pressure wave is diverging, so there are 2 terms in the derivative.

[That affects the velocity near the source.]

COMPACT SOURCES CONT'D

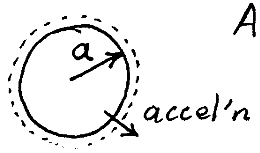
We have:

$$\boxed{\rho_0 \begin{pmatrix}\text{radial accel'n} \\ \text{of air molecules}\end{pmatrix} = \frac{K}{r^2} f(t - r/c) + \frac{K}{rc} f'(t - r/c)}$$

' indicates derivative

If the source is quite compact (radius $a << \lambda$) then the term in $\frac{1}{r^2}$ dominates over the $\frac{1}{rc}$ term.

At the source surface:



$$\rho_0 (\text{accel'n}) = \frac{K}{a^2} f(t - a/c)$$

$$\text{So, } 4\pi a^2 (\text{accel'n}) = \frac{4\pi K}{\rho_0} f(t - a/c)$$

The LHS is the source area multiplied by the acceleration of the surface, called the volume acceleration [in m^3/sec^2] of the source, we shall call it $A(t - a/c)$.

If we let $f(t - \frac{a}{c}) = A(t - a/c)$, then $K = \frac{\rho_0}{4\pi}$.

$$\text{Thus} \quad p_{\substack{\text{outside}\\ \text{source}}} = \frac{\rho_0}{4\pi r} A(t - r/c)$$

where we have allowed the volume acceleration function (defined at the source), now to be evaluated for the appropriate time delay.

VELOCITY NEAR SPHERICAL SOURCE

Since accel'n = rate of change of velocity, we can use our former equation to find velocity also, and use the proper $A(t-r/c)$.

So
$$\rho_0 \frac{dv}{dt} = \frac{\rho_0}{4\pi r^2} A(t-r/c) + \frac{\rho_0}{4\pi r c} A'(t-r/c)$$

Integrating this we have for the velocity

$$v = \frac{U(t-r/c)}{4\pi r^2} + \frac{A(t-r/c)}{4\pi r c}$$

where $U(\ )$ is the time integral of $A(\ )$, called the volume velocity, and both of them refer to the property of the compact source. Note that the second term is proportional to the pressure directly, while the first term is much larger near the source, but being a time integral, is 90° phase shifted.

Out of phase, reactive flow

$$v = \frac{U(t-r/c)}{4\pi r^2} + \frac{p}{\rho_0 c}$$

In-phase, power flow

AREA OF FLOW

(WHOOSH)

PORTION OF v OUT OF PHASE WITH FAR-FIELD PRESSURE. REPRESENTS USELESS MOVEMENT OF AIR MASS

(WHAM)

PORTION OF v THAT IS IN PHASE WITH FAR-FIELD PRESSURE, RESPONSIBLE FOR RADIATION

VELOCITY CONTINUED

For a normal harmonic (sinusoidal) wave, acceleration and velocity are related by $A = \omega U = 2\pi f U$, and the velocity phase is 90° lagging acceleration

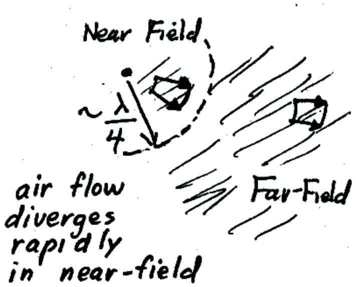
Now $\frac{1}{r}$ term is smaller than $\frac{1}{r^2}$ term near the source. Comparing them

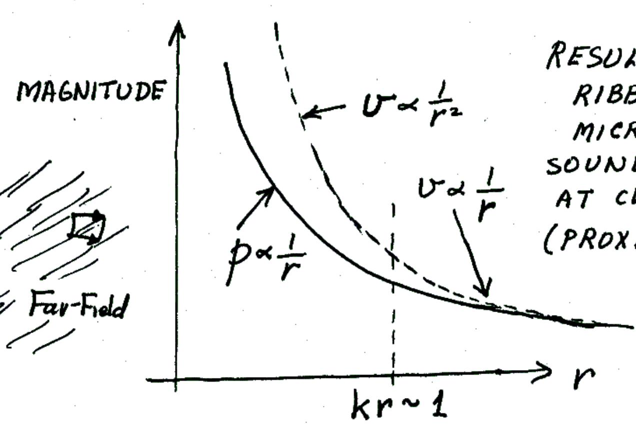
far field$_v \rightarrow \dfrac{A(\;)}{4\pi r c}$

(by letting $U(\;) = \dfrac{A(\;)}{\omega}$)

near field$_v \rightarrow \dfrac{U(\;)}{4\pi r^2} \rightarrow \dfrac{A(\;)}{4\pi r\,\omega r}$ (also phase shifted)

$$\frac{\text{near field}}{\text{far field}} = \frac{c}{\omega r} = \frac{1}{kr} = \frac{\lambda}{2\pi r} = \frac{c}{2\pi f r}$$





RESULT:
RIBBON VELOCITY MICROPHONES SOUND BASSY AT CLOSE RANGE (PROXIMITY EFFECT)

SOURCE STRENGTH

Obviously the shape of a source is not very important, if it is smaller than say $\lambda/4$, and its sound output depends on the total amount of air that it moves. In fact, it is the volume/sec², or the volume acceleration, that is proportional to the far-field pressure. For historical reasons, the source strength is made the volume/sec, or volume velocity.

$$U = \text{volume velocity} \qquad \left[m^3/s \right]$$

$$A = \text{volume accleration} \qquad \left[m^3/s^2 \right]$$

For sinusoidal fields of frequency $\omega = 2\pi f$

$$A = \omega U, \text{ and } A \text{ leads } U \text{ by } 90$$

EXAMPLE 20 cm diam sphere oscillating 1 mm peak at 100 Hz ; at 1 m :

Acceleration $a = \omega v = \omega^2 x = 4\pi^2 10^4 10^{-3}$

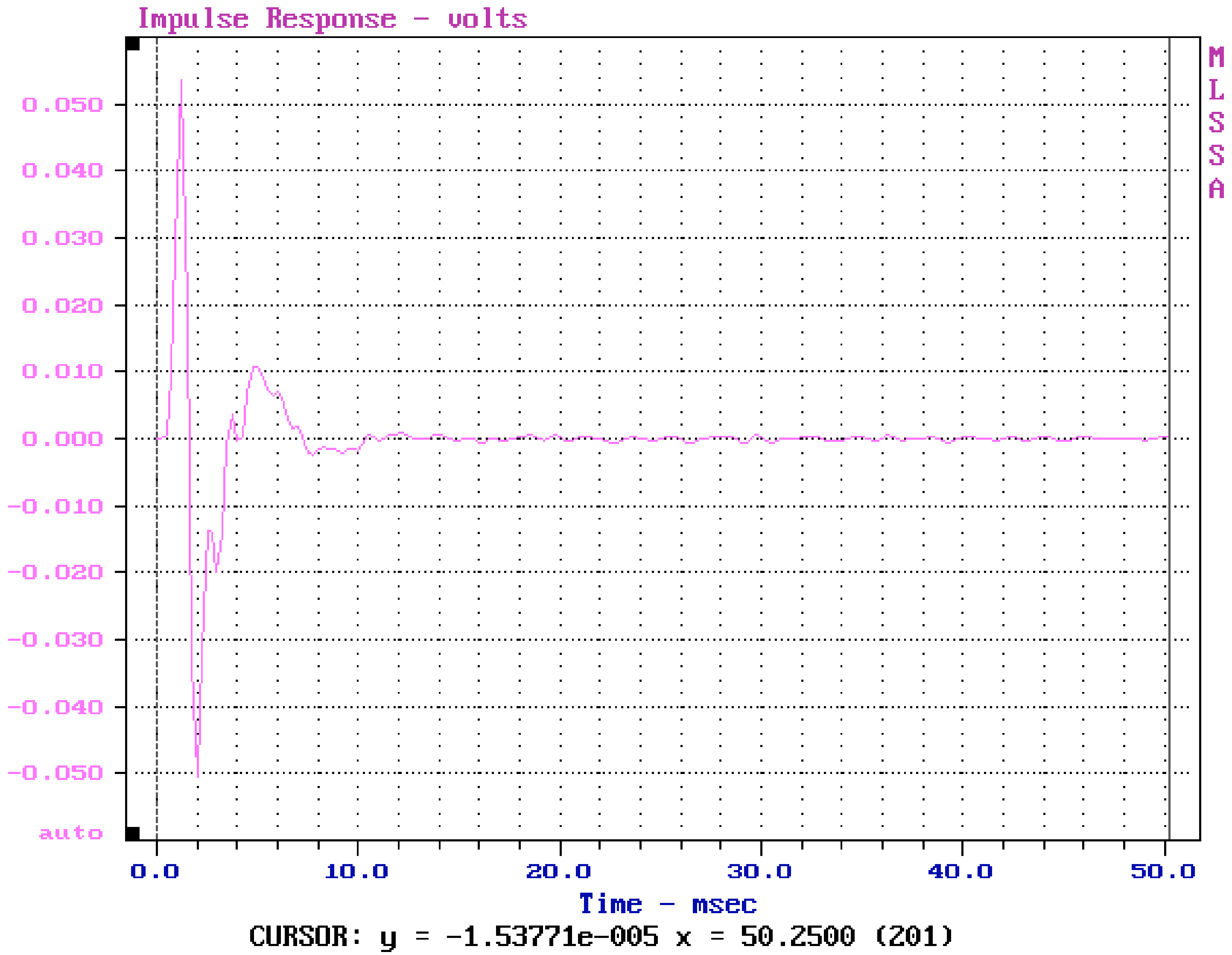
$= 394 \ m/s^2$

$$p = \frac{\rho_0}{4\pi r} A = \frac{(1 \cdot 2)\, 4\pi (0 \cdot 1)^2}{4\pi} 394$$

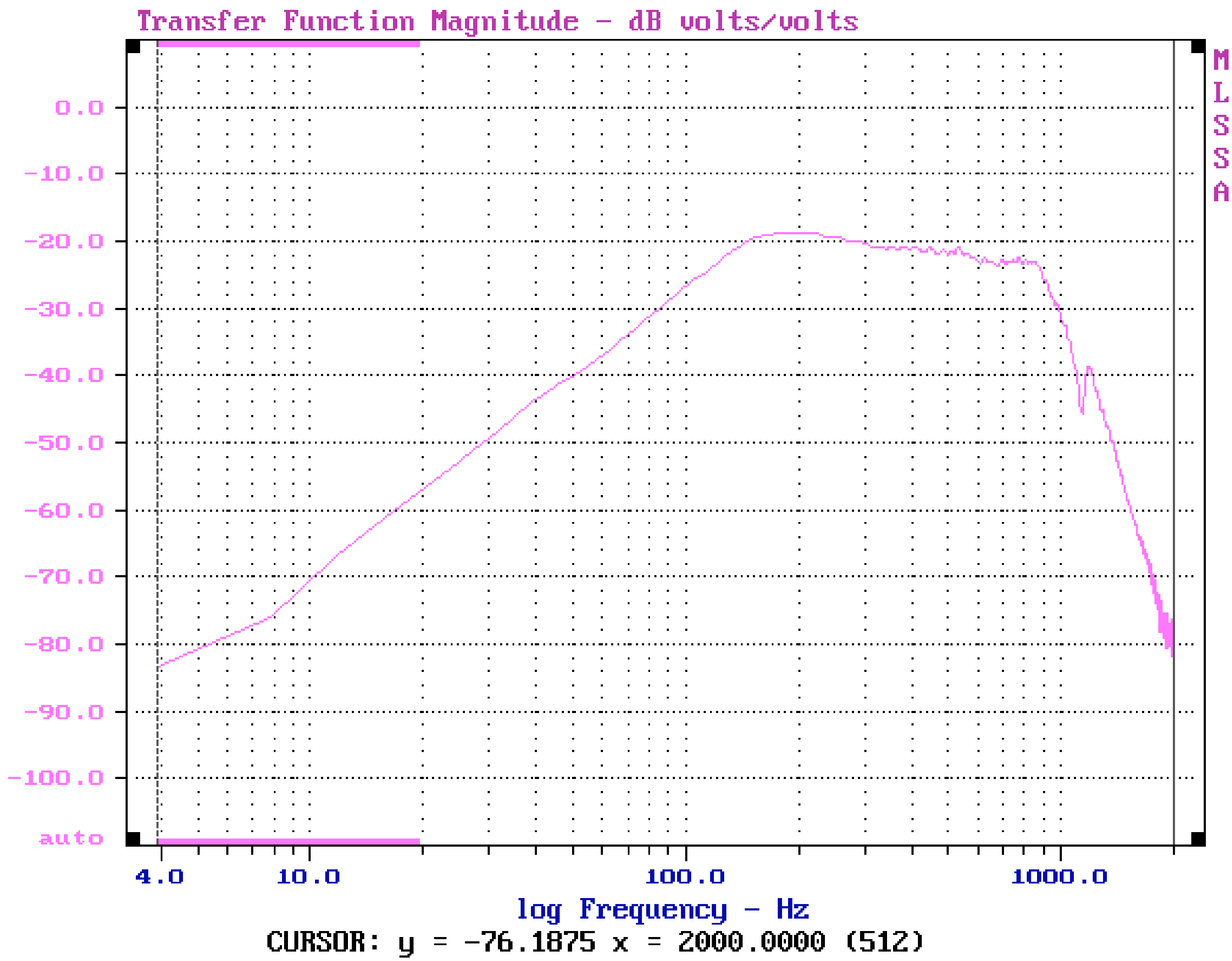
$= 4 \cdot 73 \ P_{a(peak)} \rightarrow 104 \text{ dB SPL}$

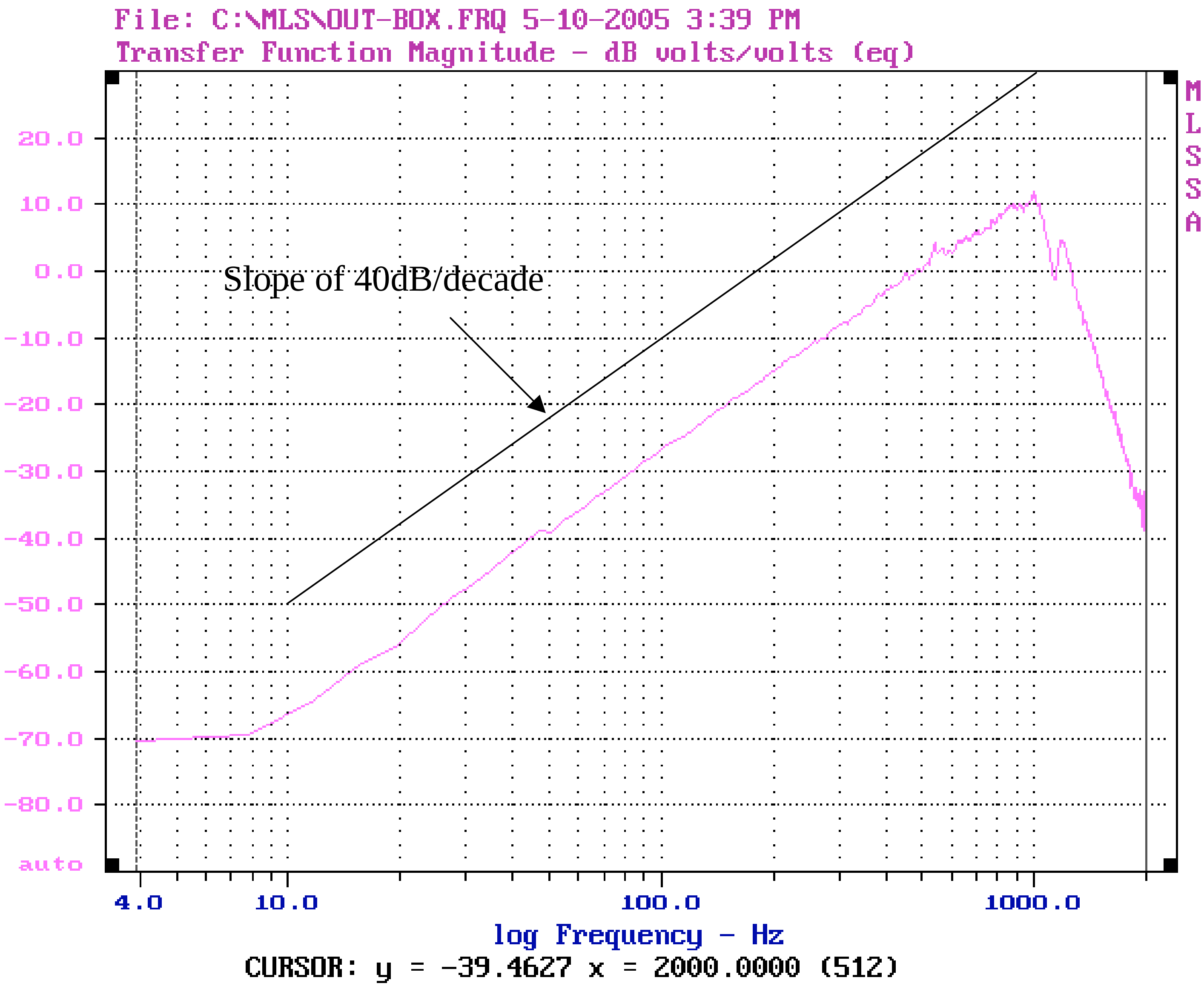
at 1kHz $\overset{\sim}{\rightarrow} 144$ dB

Measure pressure outside sealed box at dust cap



Acoustic impulse response at cone surface

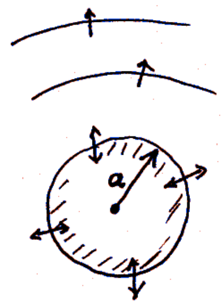

Transfer Function Magnitude - dB volts/volts
0.0
-10.0
-20.0
-30.0
-40.0
-50.0
-60.0
-70.0
-80.0
-90.0
-100.0
auto
4.0
10.0
100.0
1000.0
log Frequency - Hz
CURSOR: y = -76.1875 x = 2000.0000 (512)
MLSSA



Relative acoustic frequency response: [at cone]/[inside box]

GENERATING SPHERICAL WAVES

PULSATING SPHERE



AT SURFACE OF SPHERE $(r=a)$

$$\xi_{SOURCE} = \xi_{WAVE}$$

SUPPOSE WE MAKE

$$\xi_a = \xi_0 \sin(\omega t)$$

Then $v_a = \omega \xi_0 \cos(\omega t)$

If $ka < 1$, then there will be a whoosh in the near field, and knowing the relation between v and p versus r we can predict the pressure anywhere outside the spherical source. $\left\{ p \propto \frac{A(\ )}{r} \right\}$

If $ka \gg 1$ then $p = \rho_0 c\, v$, and the wave leaving the sphere obeys

$$\lambda \ll 2\pi a,$$

so the wave locally looks like a plane wave, and the acoustic impedance at the source surface is

$$Z = \rho_0 c \quad \text{(RESISTIVE)}$$

IMPEDANCE, INTENSITY

BY USING THE NEAR & FAR-FIELD COMPONENTS OF v, WE HAVE

$$v = \frac{p}{\rho_o c} + \frac{p}{jkr\rho_o c}$$

reactive term

real power

GIVING $\frac{p}{v} = Z = \frac{\rho_o c}{1 + 1/jkr} = \frac{jkr}{1+jkr}\rho_o c$

(like 1st order high-pass)

MEANING – given v (or U), find p

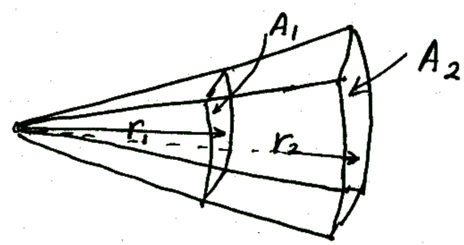
AT LOW kr (<1), $p \simeq jkr\rho_o c\, v$

$$\simeq j\omega v (4\pi r^2) \frac{\rho_o}{4\pi r}$$

It was shown that for a compact source $p = \frac{\rho_o}{4\pi r} A(t - r/c)$

volume accl'n

$$\text{Intensity} = \frac{p^2}{\rho_o c} \propto \frac{1}{r^2}$$



Power thru A_1, A_2

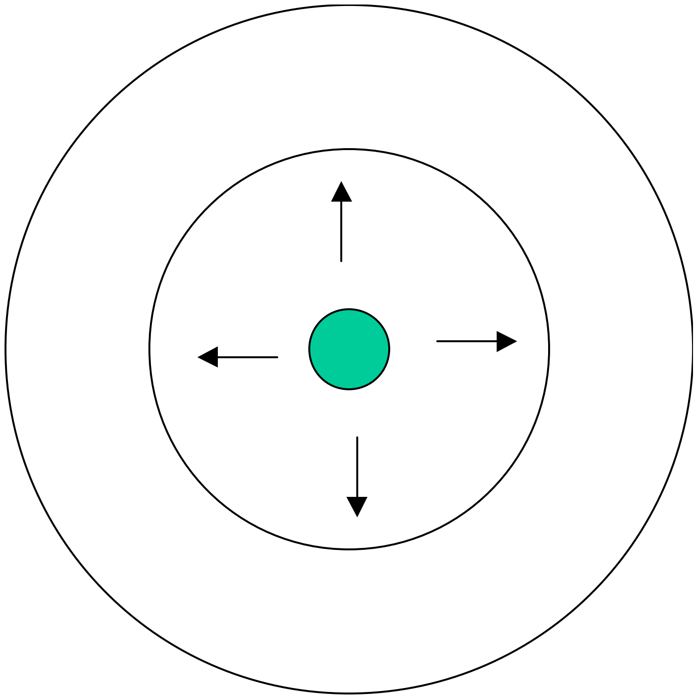
$$A_1 \frac{p_1^2}{\rho_o c} = A_2 \frac{p_2^2}{\rho_o c}$$

SO THE WAVE ENERGY IS CONSERVED

$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$$

Shows $p=f(r,t)/r$

However, for a 3-dimensional spherically-spreading wave from a point source, the amplitude of the harmonic solution to the wave equation for the pressure [1,2] can be written as



$$p(r,\omega) = \rho\, j\omega\, U \exp(-jkr)/(4\pi r),$$

where U is the volume velocity [m^3/s] of the point source. The $j\omega$ factor represents a time derivative, and we can thus write the solution in the time domain as

$$p(r,t) = \rho\, A(t-r/c)/(4\pi r),$$

where A(..) is the volume acceleration [m^3/s^2] of the source. Note that the solution for the pressure does not change shape as it propagates, but the amplitude falls off as 1/r.

The particle velocity, v, relates to the pressure by the Newtonian equation of motion for the air

$$\nabla p = -\rho\ \partial v/\partial t.$$

Mathematically, for a spherically-symmetric wave solution, $\nabla p = \partial p/\partial r$, and thus

$$\nabla[\exp(-jkr)/r] = -jk\ \exp(-jkr)/r - \exp(-jkr)/r^2.$$

As a result, the Newtonian equation relates the pressure to the particle velocity of the air, at radius r, as

$$(1 + 1/jkr)\ p = \rho\ c\ v,$$

which can also be written as

$$(jkr + 1)\ p = jkr\ \rho\ c\ v = \rho\ j\ \omega\ r\ v = \rho\ a\ r,$$

where a is the acceleration of the air particles.

Plot of Z = R+jωM for a sphere.

For a sphere, the acoustic impedance is easy to work out. For kr<<1, the size of the sphere is much less than a wavelength, and the shape of the source is not very important.

Thus we expect all acoustic impedances of monopole acoustic sources to act similarly.

$\underbrace{R \propto f^2}$ agrees with volume acceleration

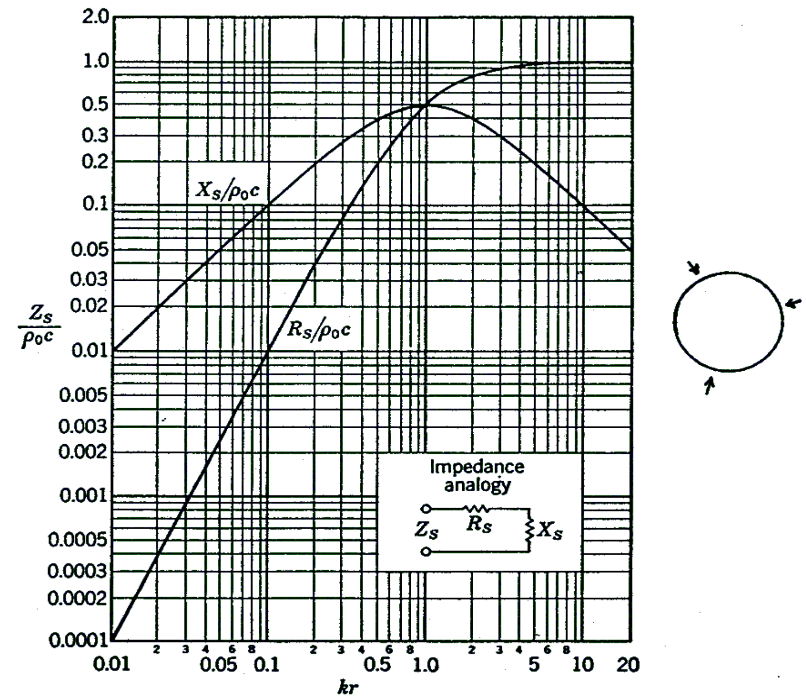


FIG. 2.10. Real and imaginary parts of the normalized specific acoustic impedance $Z_s/\rho_0 c$ of the air load on a pulsating sphere of radius r located in free space. Frequency is plotted on a normalized scale where $kr = 2\pi f r/c = 2\pi r/\lambda$. Note also that the ordinate is equal to $Z_M/\rho_0 cS$, where Z_M is the mechanical impedance; and to $Z_A S/\rho_0 c$, where Z_A is the acoustic impedance. The quantity S is the area for which the impedance is being determined, and $\rho_0 c$ is the characteristic impedance of the medium.

The rising low-frequency imaginary part represents the reasonably constant inertial air load.

Note the curve is very smooth with no resonances or interference. That is true since diffraction from edges does not occur for a sphere.

Piston in Infinite Baffle

The LF imaginary part of the acoustic impedance is proportional to frequency, representing a constant inertial air load.

The real part is proportional to frequency^2, which is expected for a monopole source.

The oscillations relate to interference from edge reflections.

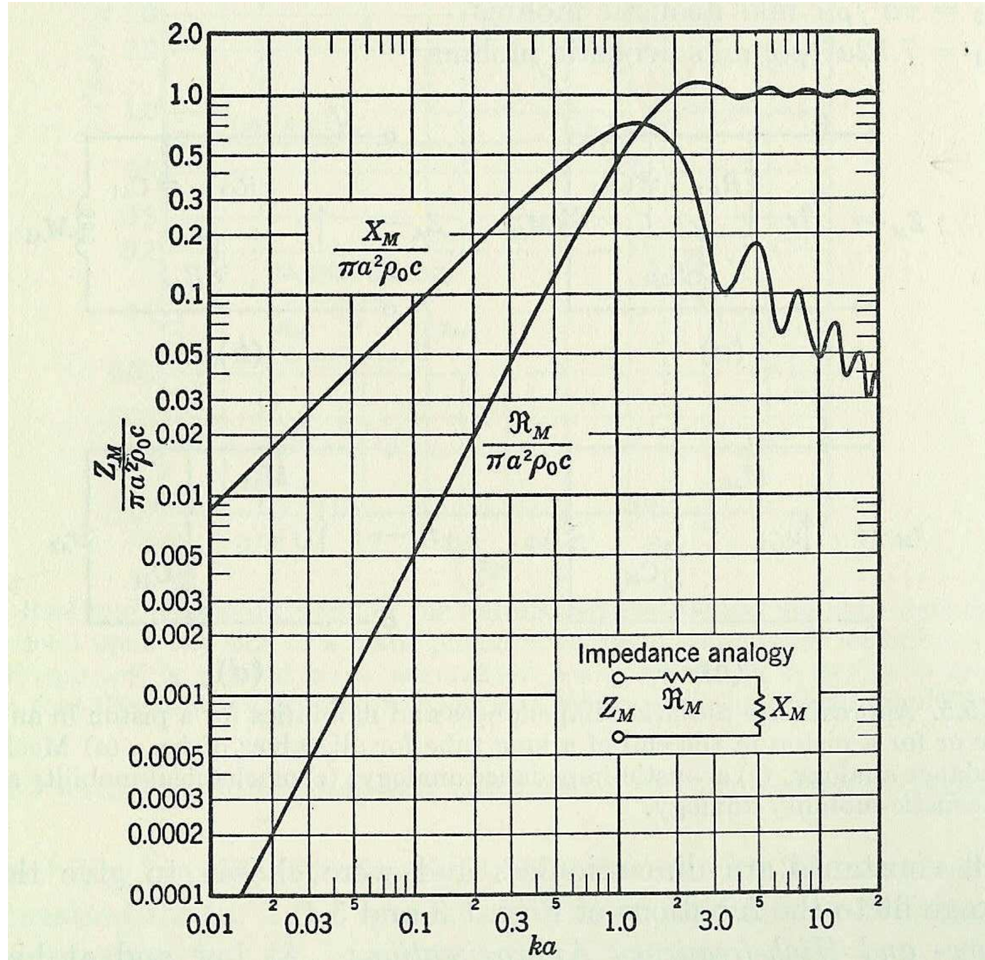
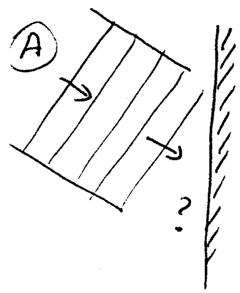


Figure 1. Showing the real and imaginary parts of the acoustic surface impedance for a piston in an infinite baffle, versus ka, where a is the piston radius. After Beranek.

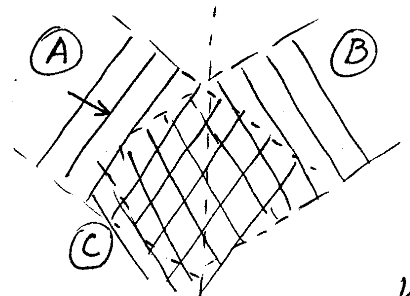
PROPAGATION

HARD SURFACES

AIR CAN MOVE ALONG A SURFACE, BUT NOT INTO IT. AT THE SURFACE THE PRESSURE CAN BUILD UP, BUT PERPENDICULAR VELOCITY = 0. SO SOUND MUST BE CONSISTENT WITH THIS BOUNDARY CONDITION.



REAL SITUATION



SIMULATION WITHOUT WALL

B IS A WAVE WHICH PRODUCES AT THE ORIGINAL WALL POSITION THE PROPER BOUNDARY CONDITION

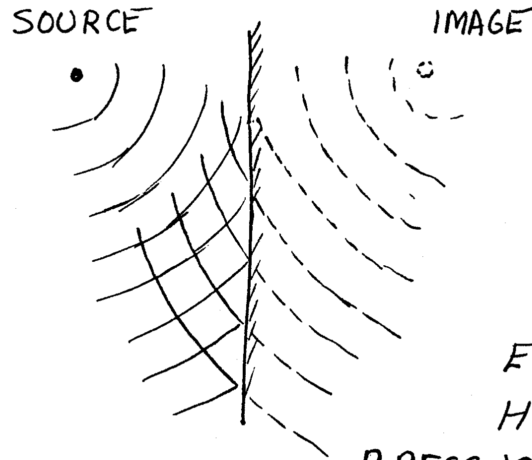
BECAUSE OF SYMMETRY THE NET VELOCITY LIES ALONG THE DIRECTION OF THE WALL

NET VELOCITY

THE WAVE C IS A CONTINUATION OF B, AND REPRESENTS REFLECTION IN THE REAL SITUATION

NOTE THAT THE <u>PRESSURE</u> AT THE WALL IS <u>DOUBLED</u> FROM THAT EXPECTED FROM A ALONE.

IMAGE CONCEPTS



THE IMAGE HERE ENSURES THAT AT EACH POINT ON THE WALL THE PARTICLE VELOCITY IS ALONG THE WALL, EVEN THOUGH THE WAVES HAVE CURVED WAVEFRONTS.

PRESSURE ON WALL IS DOUBLED.

A COMPLEX SOURCE WORKS THE SAME WAY.
THE IMAGE HAS MIRROR SYMMETRY

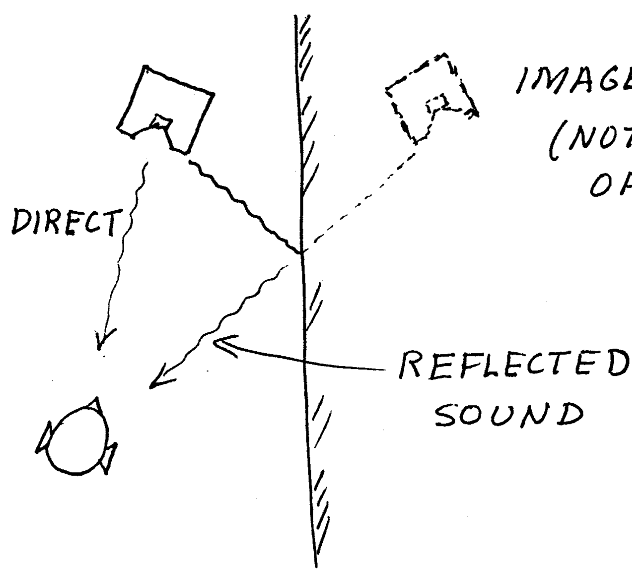
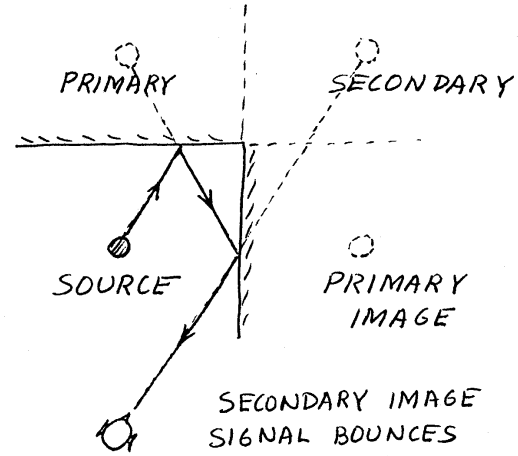
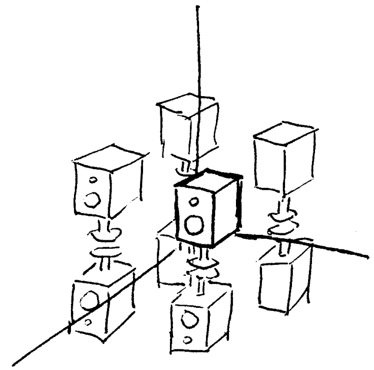


IMAGE
(NOTE THE HIGH OFF-AXIS ANGLE OF THIS SIGNAL)

MULTIPLE REFLECTIONS, ROOMS



SECONDARY IMAGE
SIGNAL BOUNCES
OFF TWO WALLS



7 LOUDSPEAKER IMAGES

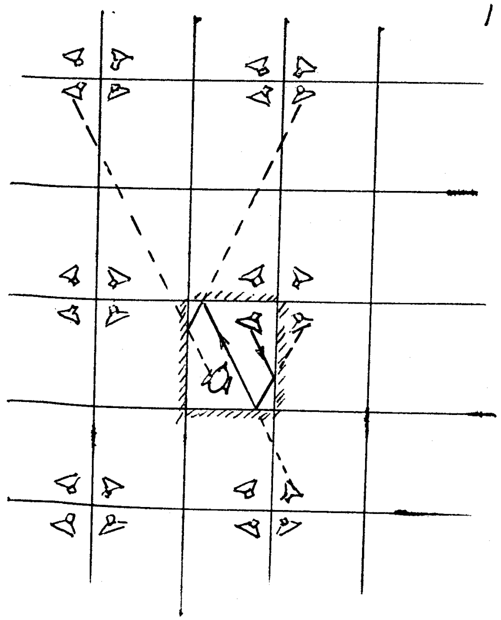
AT VLF

8 COHERENT SOURCES

→ 3×6 → +18 dB ← At bass frequencies

IF POWERS ADD, +9 dB

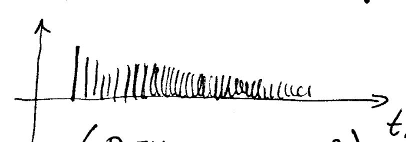
PATH OF AN OBLIQUE RAY



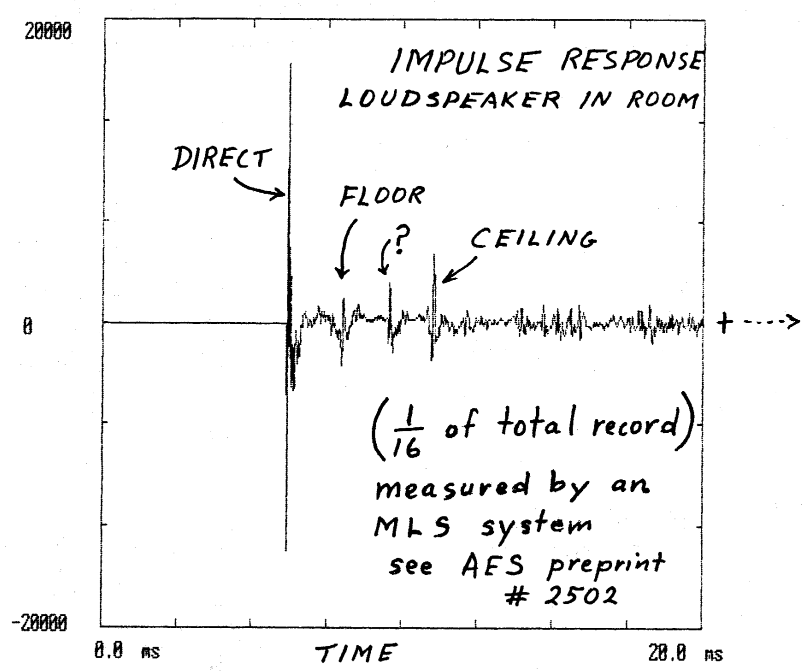
IN TIME t, SOUND TRAVELS ct, AND ALL REFLECTIONS IN SPHERE RADIUS ct ARRIVE.

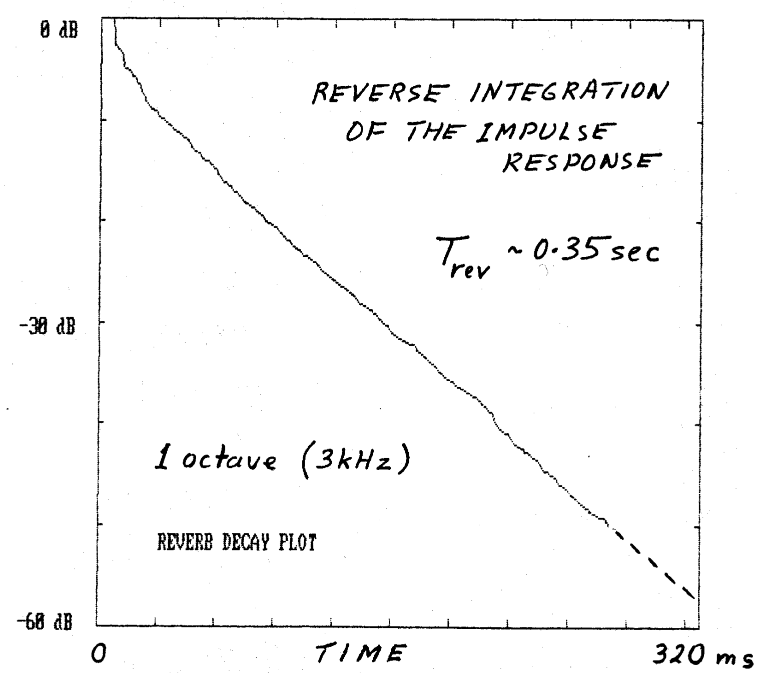
REFLECTIONS $\propto (ct)^3$

IMPULSE RESPONSE:



(DENSITY $\propto t^2$)


20000
0
-20000
IMPULSE RESPONSE
LOUDSPEAKER IN ROOM
DIRECT
FLOOR
?
CEILING
(1/16 of total record)
measured by an
MLS system
see AES preprint
2502
0.0 ms
TIME
20.0 ms


0 dB
-30 dB
-60 dB
REVERSE INTEGRATION
OF THE IMPULSE
RESPONSE
T_{rev} ~ 0.35 sec
1 octave (3kHz)
REVERB DECAY PLOT
0
TIME
320 ms

Listen to baffle…

From Olson *Acoustics*

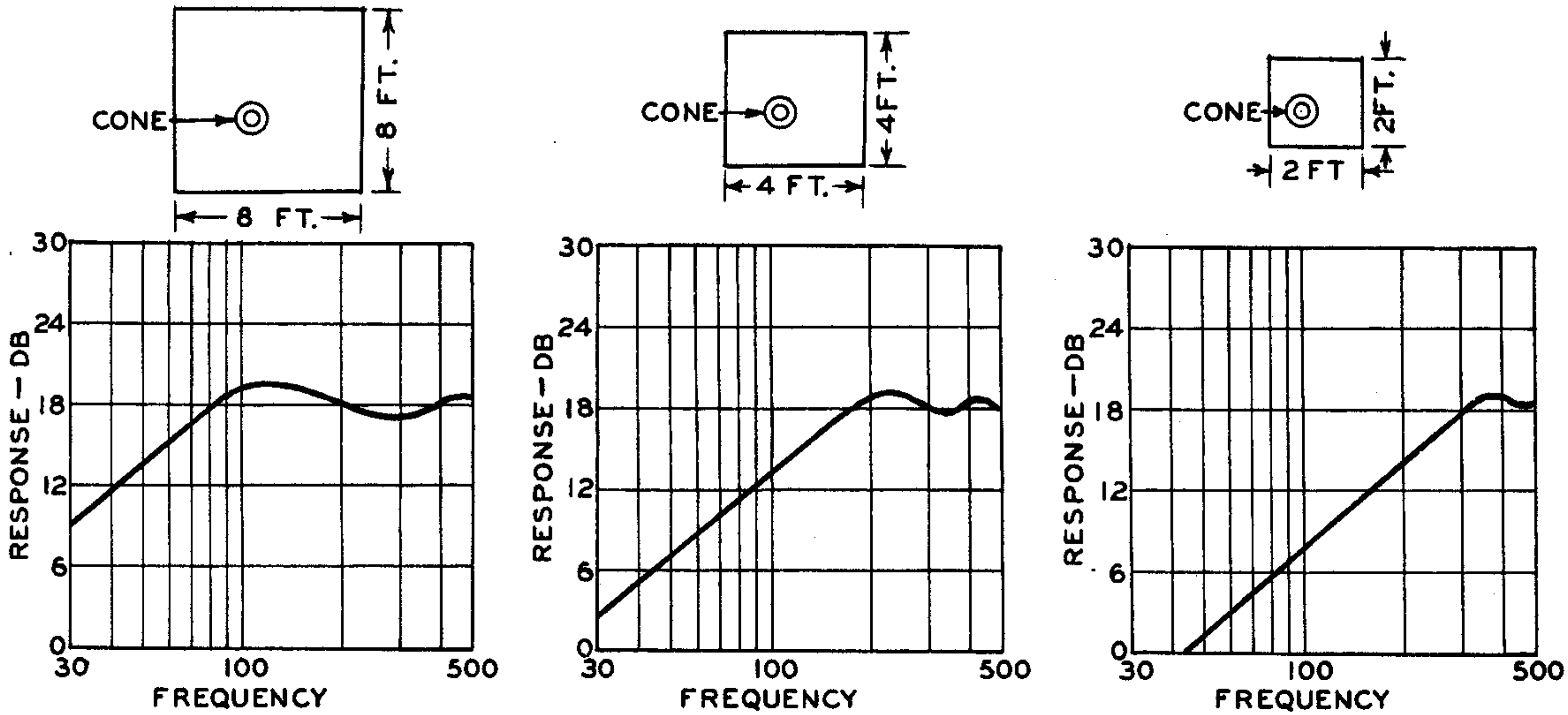
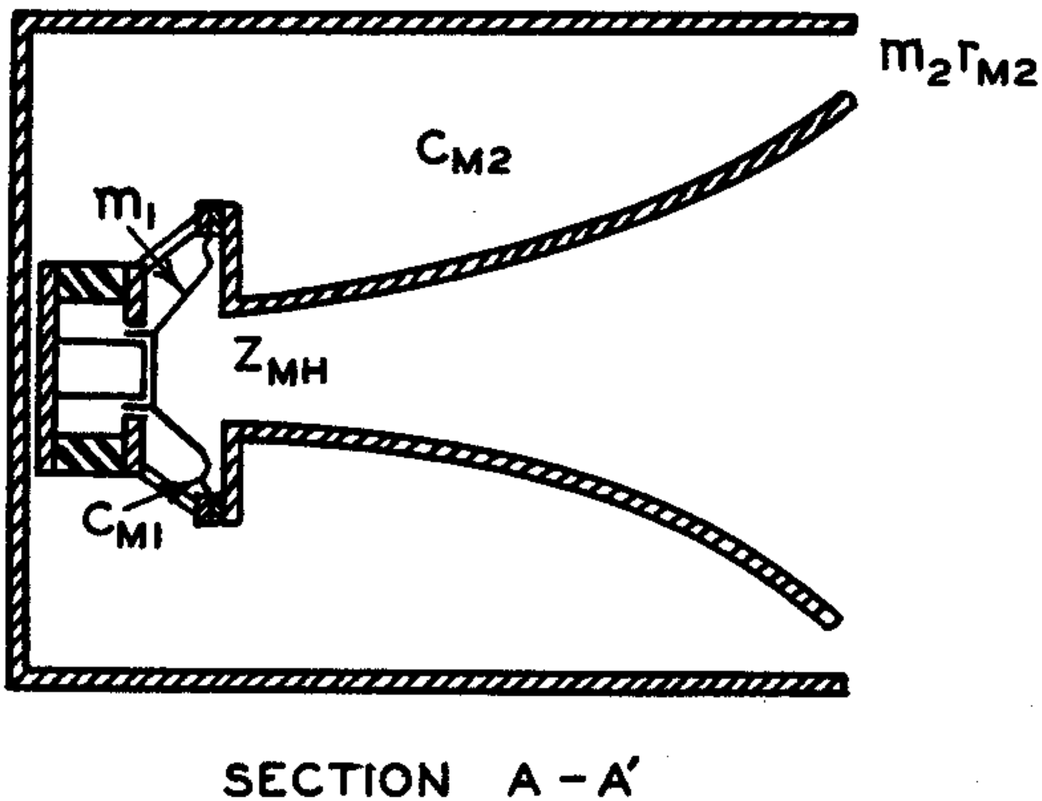


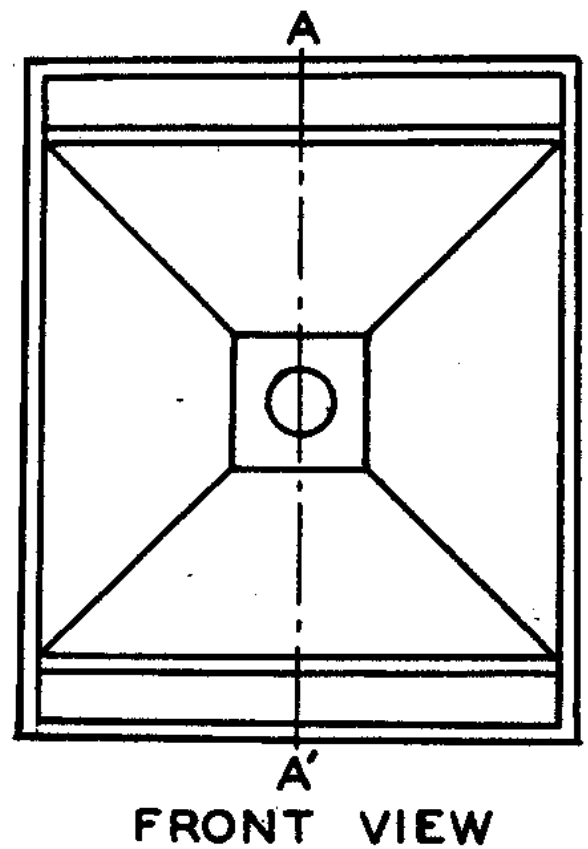
FIG. 6.22. Pressure response frequency characteristics of mass-controlled, direct radiator, dynamic loudspeaker mechanisms, with 10-inch diameter cones, mounted in square baffles of 8, 4, and 2 feet on a side.

Conclusion: we need a huge baffle to give decent bass

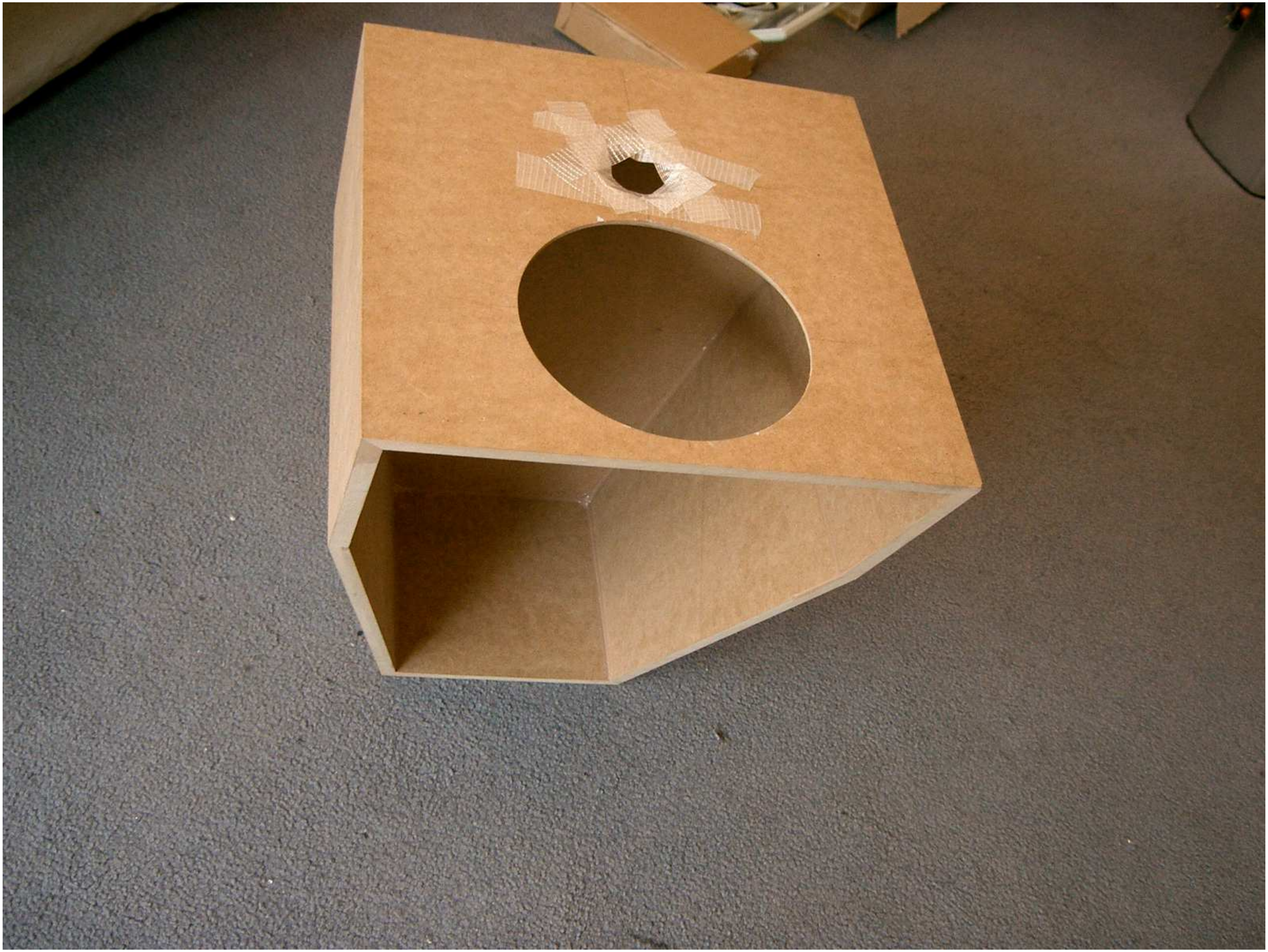
Horn design from Olsen

Horns are the most efficient speakers,
but often give a coloured response.

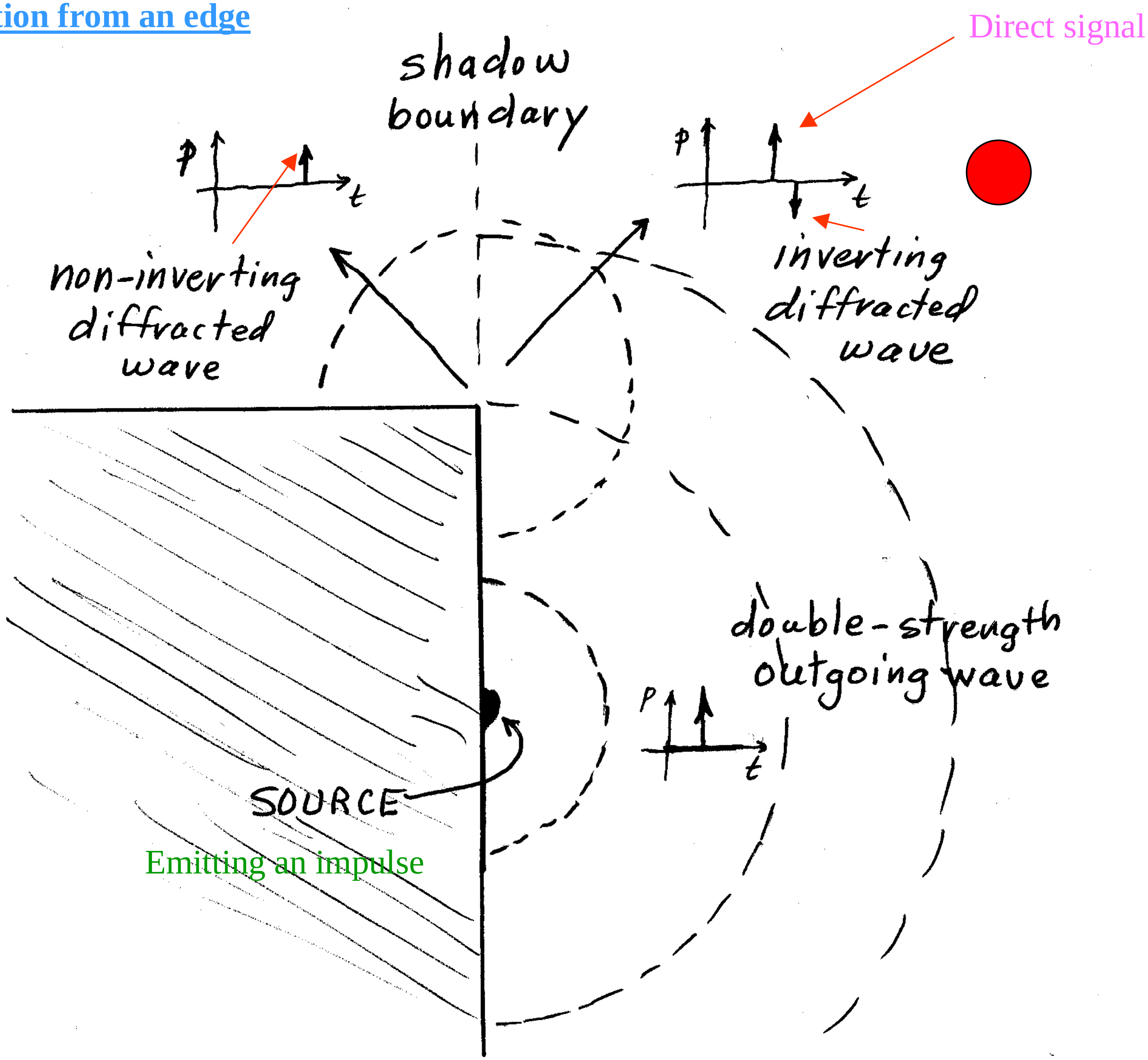


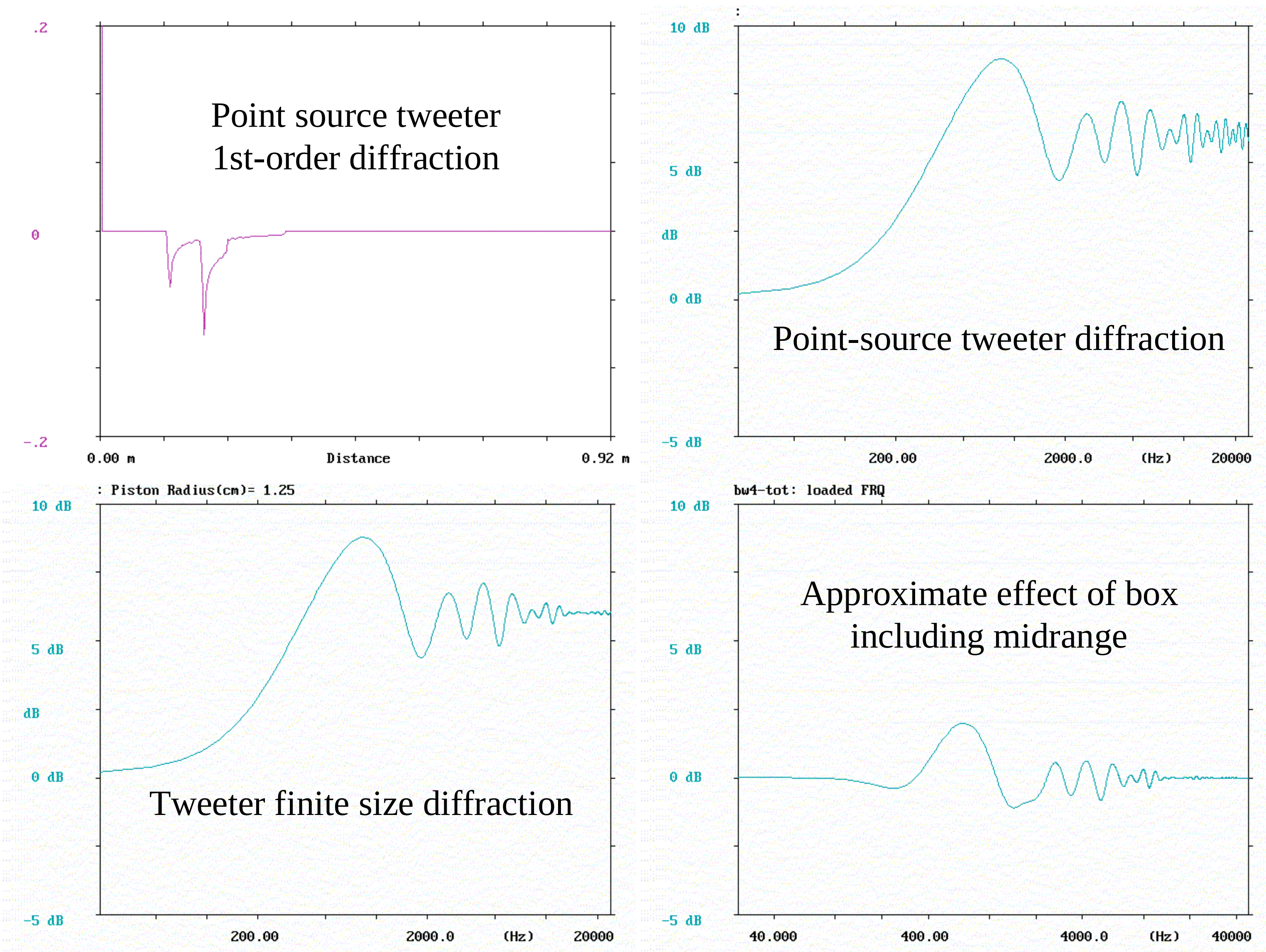


The new “robot” look
for the 800 series



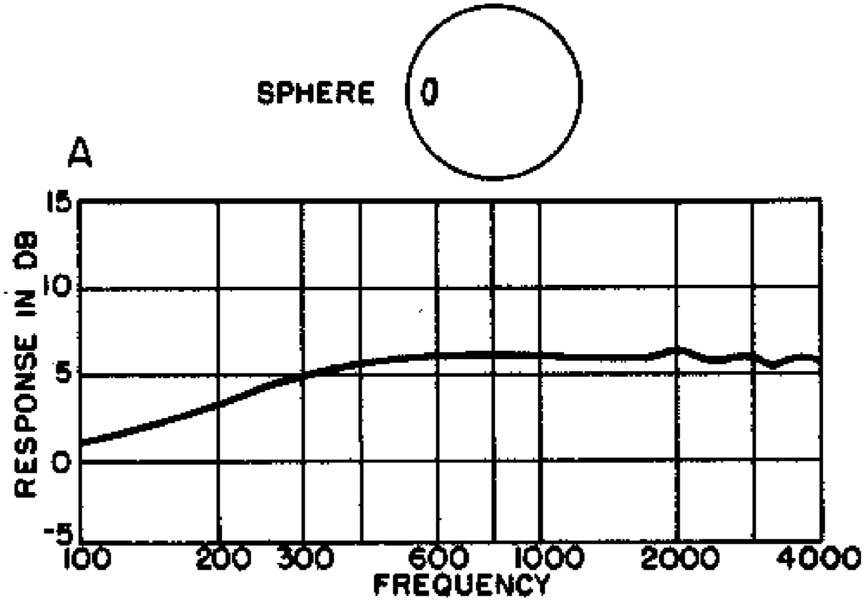
Diffraction from an edge

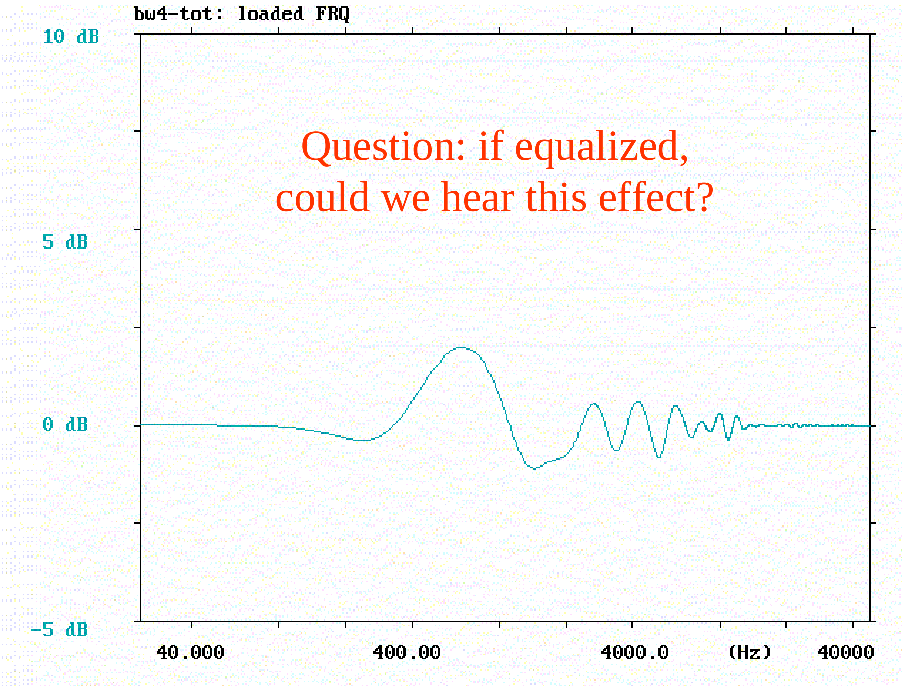




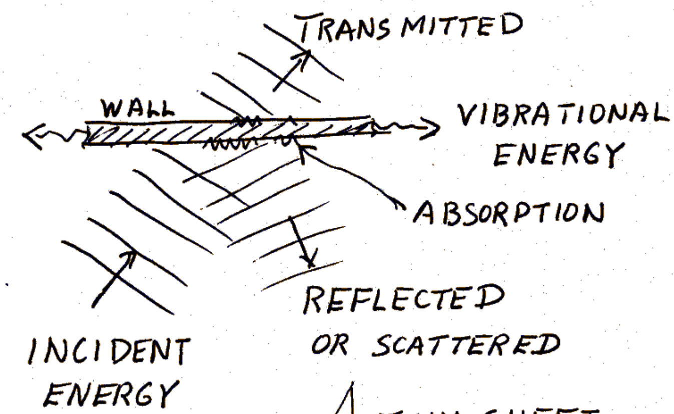
Diffraction calculations using a simple model

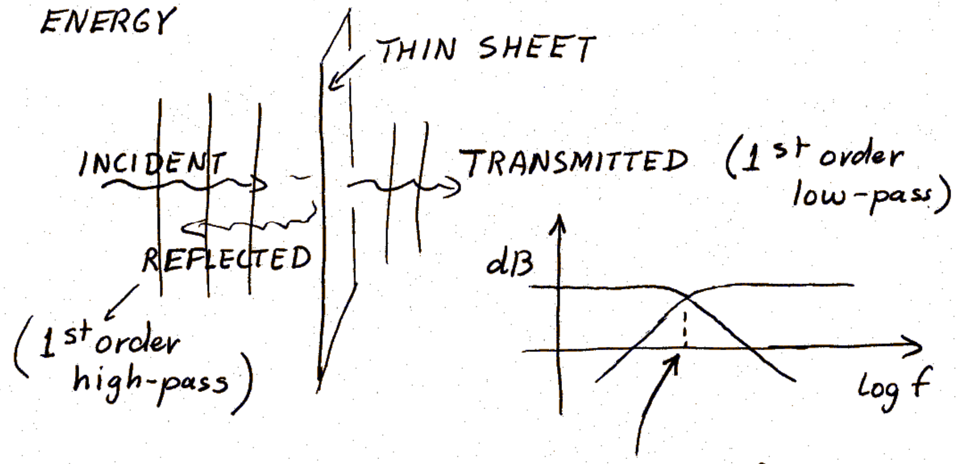

B&W


SPHERE
0
A
RESPONSE IN DB
15
10
5
0
-5
100
200
300
600
1000
2000
4000
FREQUENCY


bw4-tot: loaded FRQ
10 dB
5 dB
0 dB
-5 dB
40.000
400.00
4000.0
(Hz)
40000
Question: if equalized,
could we hear this effect?

VIBRATION, TRANSMISSION



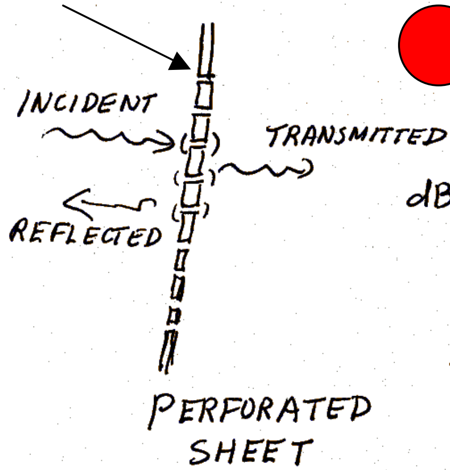


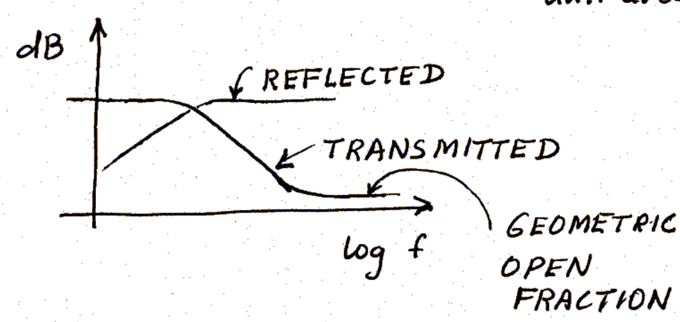
At corner frequency :

$$\frac{\text{mass of sheet}}{\text{unit area}} = \frac{1}{\pi}\left(\frac{\text{mass of } \lambda \text{ of air}}{\text{unit area}}\right)$$

For paper,
f ~ 1.7kHz

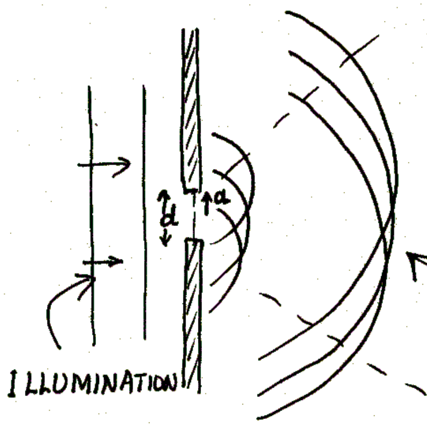
Movie screen





HUYGEN'S PRINCIPLE

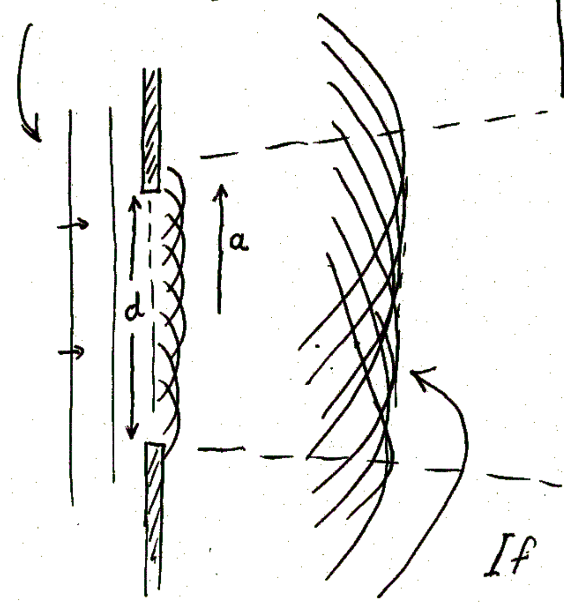
"EACH POINT ON A WAVEFRONT ACTS AS A NEW SOURCE OF SPHERICAL WAVES"



If $d < \frac{\lambda}{2}$ then the diffracted wave disperses well. $[ka < 1.5]$

(like a spherical wave)

$$\boxed{ka = 1.5 \rightarrow f \cong \frac{c}{4a}}$$



If $d >> \frac{\lambda}{2}$ then the aperture will emit a beam. $[ka > 1.5]$

(like a plane wave)

EXAMPLE

20 cm driver

If freq < 860 Hz it disperses

For freq > 860 Hz it becomes directional.

Directivity relates to diffraction

DIRECTIVITY OF A PISTON IN A BAFFLE

This summation appears in many texts[6] and, for the case of r large compared with the radius of the piston a, leads to the equation

$$p(r,t) = \frac{\sqrt{2}\, jf\rho_0 u_0 \pi a^2}{r} \left[\frac{2J_1(ka \sin\theta)}{ka \sin\theta}\right] e^{j\omega(t-r/c)}$$

where u_0 = rms velocity of the piston

$J_1(\ )$ = Bessel function of the first order for cylindrical coordinates[6]

ka = 1 means

550 Hz for a = 10cm

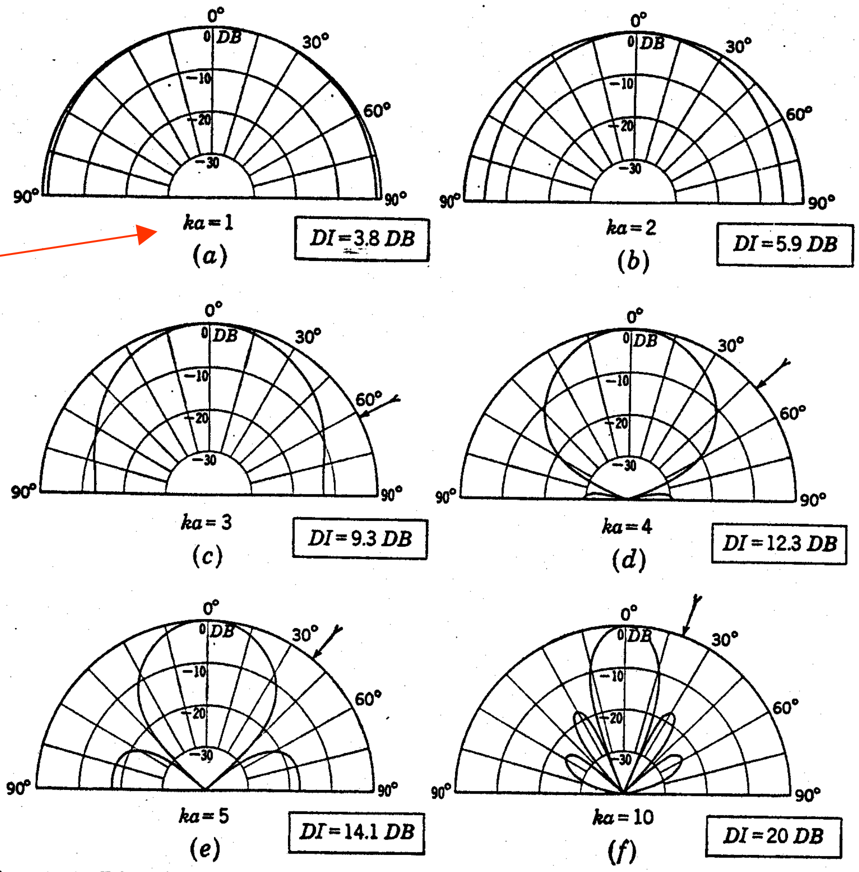
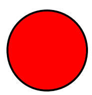


FIG. 4.10. Directivity patterns for a rigid circular piston in an infinite baffle as a function of $ka = 2\pi a/\lambda$, where a is the radius of the piston. The boxes give the directivity index at $\theta = 0°$. One angle of zero directivity index is also indicated. The DI never becomes less than 3 db because the piston radiates only into half-space.

DIRECTIVITY OF PISTON IN A LONG TUBE

This shows that a
small source will
be omnidirectional
if size << wavelength



ka = 1 means
550 Hz for a = 10cm

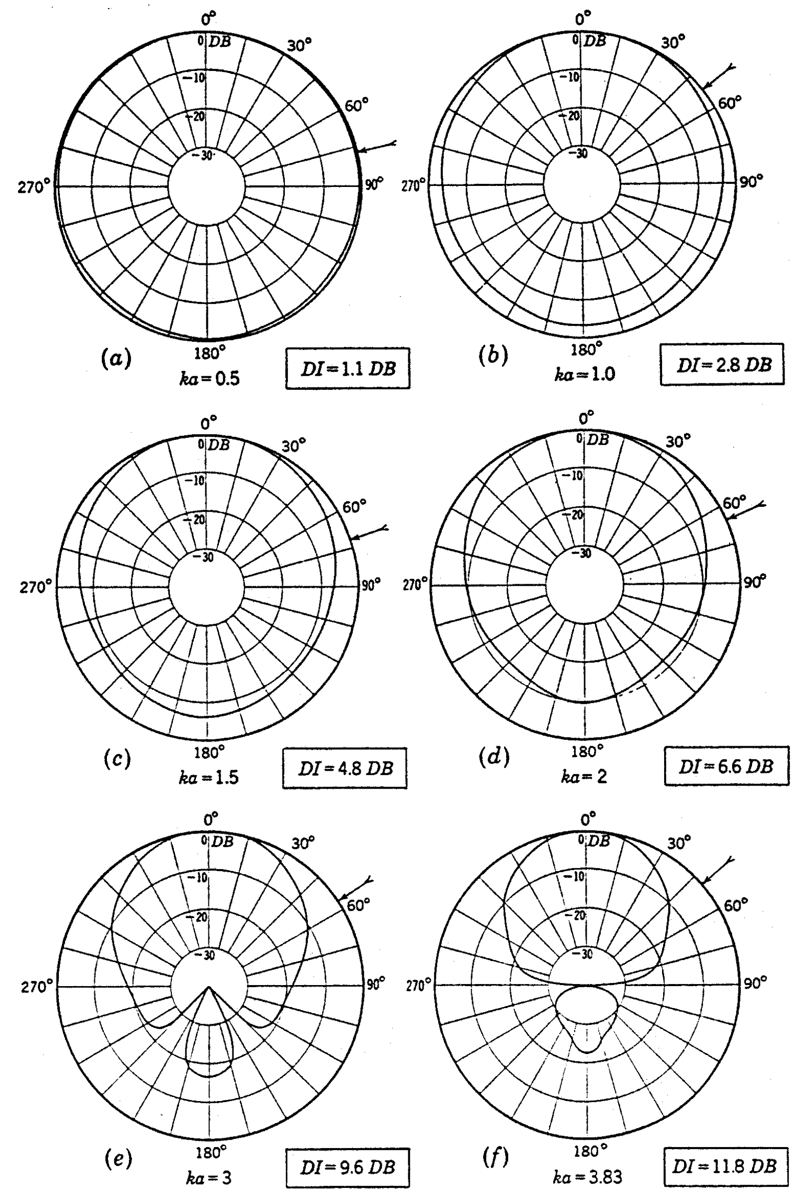
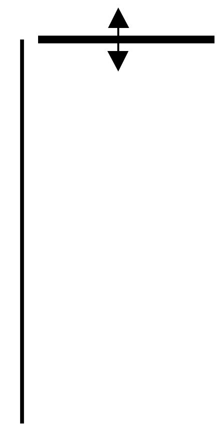
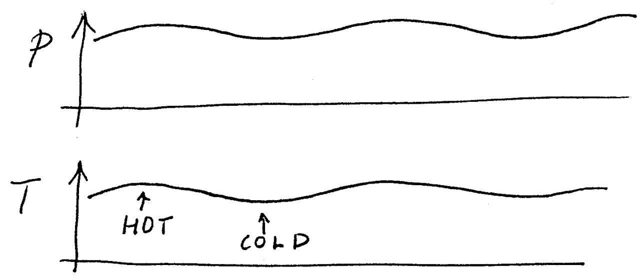


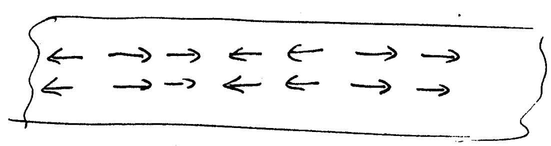
FIG. 4.12. Directivity patterns for a rigid circular piston in the end of a long tube as a function of $ka = 2\pi a/\lambda$, where a is the radius of the piston. The boxes give the directivity index at $\theta = 0°$. One angle of zero directivity index is also indicated.



WHY SOUND DECAYS

As a sound wave moves, pressure and temperature variations occur.
One mechanism for energy decay is heat flow due to temp. variation.





HEAT FLOW PATTERN

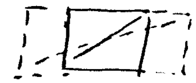
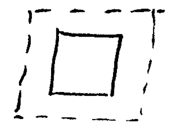
Now Heat Flow $\propto \dfrac{\Delta T}{\Delta x} \propto \dfrac{1}{\lambda} \propto f$
(or heat current)

The loss is slight, and acts like loss in a resistor. Power loss $= I^2 R$

HERE $\dfrac{\text{ENERGY LOSS}}{\text{TIME}} \propto f^2$

At lower frequencies, we might expect a longer heat flow time would increase the loss, but at higher frequencies the gradient is higher, and more important.

OTHER DECAY MECHANISMS

- Thermal Conduction (already)
- Viscous shear 
- Bulk viscosity 

picture of the deformation

Organized molecular motion (sound) is turned into heat by the above mechanisms. At a material boundary the viscous and thermal effects are enhanced, and a loss per bounce $\propto$ freq. occurs.

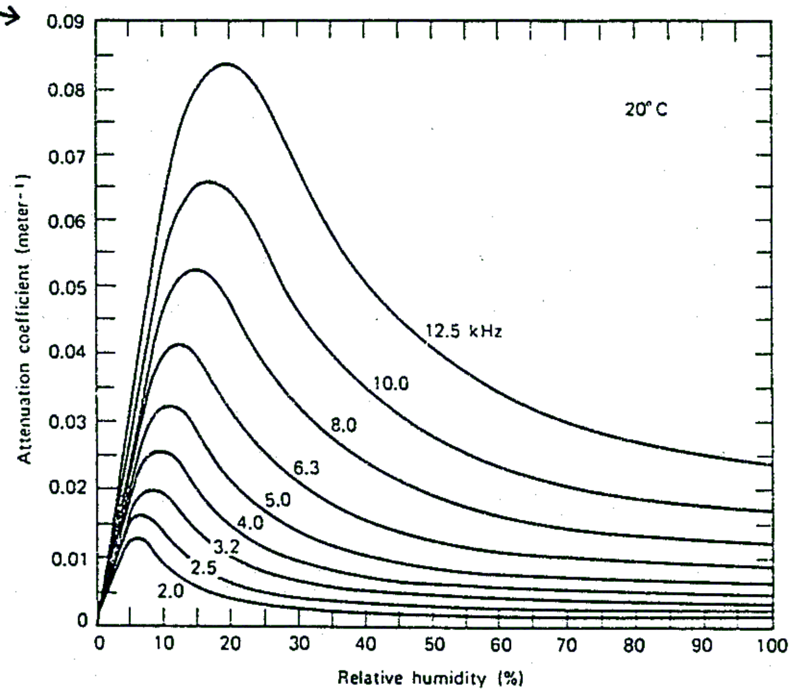
In addition, gas molecules undergo vibrational modes with long relaxation times ($\sim 10^{-3}$ sec) and the presence of water vapour affects this relaxation so humidity affects the attenuation of sound.

All of the above effects will combine into a term labeled *total attenuation coefficient* and designated by the letter m. This term is frequency, temperature, and humidity dependent. For the case of a plane traveling wave, the following relationship holds :

$$P = P_0 e^{-mx/2}$$

where P_0 is the pressure amplitude at distance $x = 0$,

almost 0.4 dB/m



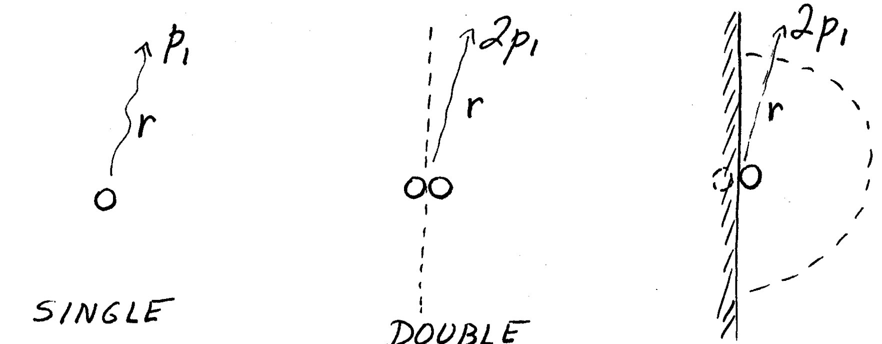
Ultrasonic frequencies decay rapidly

Total attenuation coefficient m versus relative humidity for air at 20°C (68°F) as a function of frequency.

from D. Bohn JAES 36 April 1988 p229

GENERATION OF SOUND

POINT SOURCE ON INFINITE BAFFLE



SINGLE POINT SOURCE

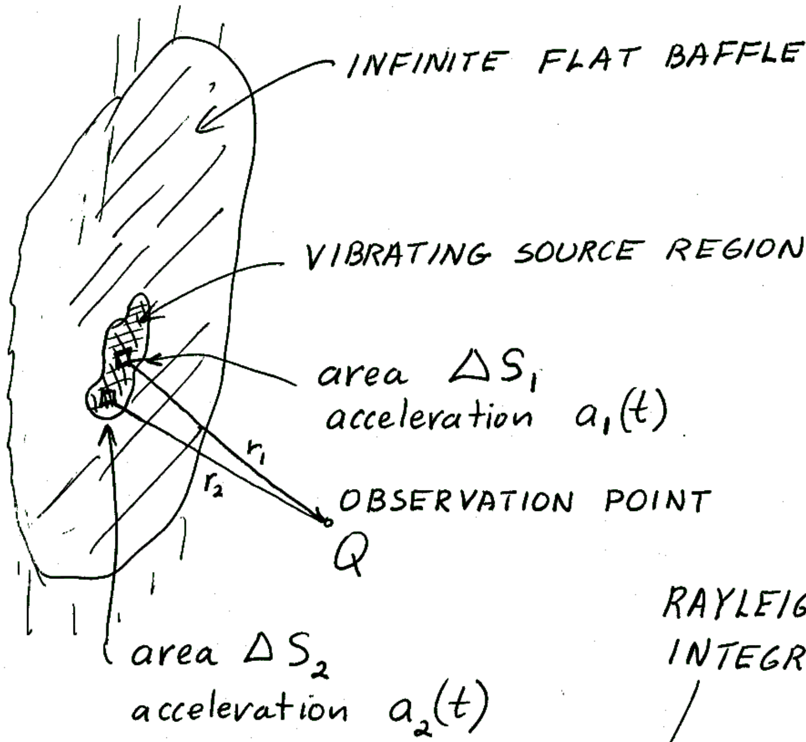
DOUBLE SOURCE (very close)

SINGLE SOURCE ON BAFFLE AND PERFECTLY SPHERICAL WAVES

$p = \dfrac{\rho_0 A(t - r/c)}{4\pi r}$	$p = \dfrac{\rho_0 A(t - r/c)}{2\pi r}$ [double]

The actual source can be considered a sum of sources acting as points. This superposition principle works if the sources don't interact, and this is fairly valid if $\dfrac{v_{SOURCE}}{c}$ = MACH No. $\ll 1$

EXTENDED SOURCE ON INFINITE BAFFLE



RAYLEIGH INTEGRAL SUM

PRESSURE AT Q

$$p(t) = \frac{\rho_0 \, \Delta S_1 \, a_1(t - r_1/c)}{2\pi r_1} + \frac{\rho_0 \, \Delta S_2 \, a_2(t - r_2/c)}{2\pi r_2} + \cdots$$

The division of the source elements should be much smaller than λ, where λ is the highest frequency to be considered.

RADIATION IMPEDANCE
PISTON IN INFINITE BAFFLE

This slope represents a mass of air of 0.85 x radius

The "air load"

$\rho_0 c$ ---

$X \propto f$

Radiation impedance = pressure/velocity

$R \propto f^2$

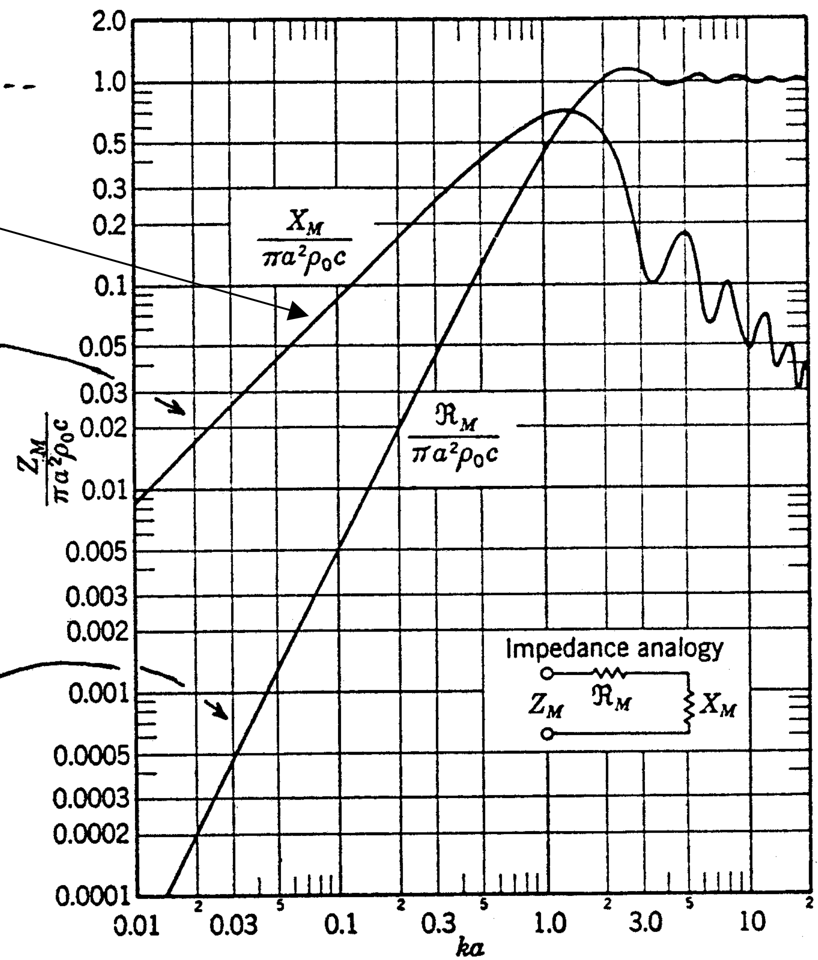
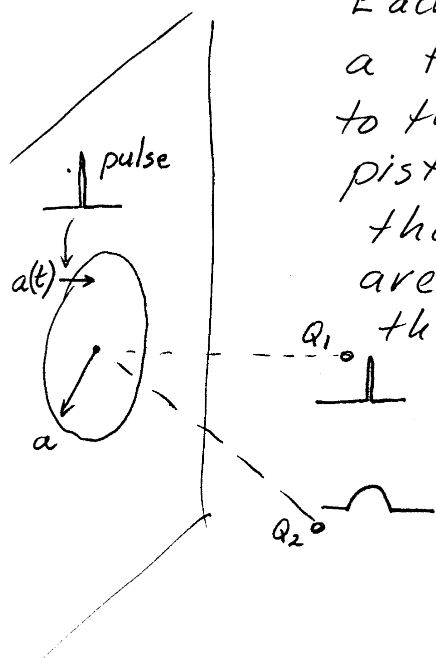


FIG. 5.3. Real and imaginary parts of the normalized mechanical impedance ($Z_M/\pi a^2\rho_0 c$) of the air load on one side of a plane piston of radius a mounted in an infinite flat baffle. Frequency is plotted on a normalized scale, where $ka = 2\pi fa/c = 2\pi a/\lambda$. Note also that the ordinate is equal to $Z_A \pi a^2/\rho_0 c$, where Z_A is the acoustic impedance.

PISTON IN INFINITE BAFFLE



Each elemental source has a far-field pressure proportional to the acceleration of the piston. For frequencies low enough that $\lambda > 4a$, all the sources are roughly in phase, and the whole piston acts like a point source.

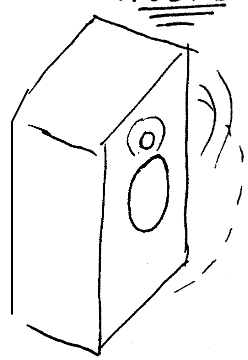
If we apply a pulse of acceleration to the piston, each source element gives off a pulse pressure wave of sound.

On axis, the result will be a short pulse. Off axis, this pulse will spread due to transit time differences.

<u>Example</u> 20 cm driver, 45° off axis. Path differences are 20 sin 45° cm. giving a time spread of ~0.4 ms. This is short enough to give a flat response to ~ 1 kHz.

(20 cm) 8 inch driver example

WOOFER



Piston (or cone) oscillates with peak of ±1 mm at 100 Hz. At 1 m:

$$p = \frac{\rho_0}{2\pi r} \text{(volume acceleration)}$$

(on baffle)

$$= \frac{\rho_0}{2\pi r} \pi a^2 \omega^2 x$$

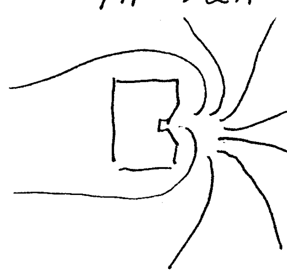
$$\boxed{\text{At very low frequency, we should use } 4\pi.}$$

$$= \frac{(1.2)\,\pi (0.1)^2 4\pi^2\, 10^4\, 10^{-3}}{2\pi}$$

"$4\pi \rightarrow 2\pi$ transition"

(monopole)

see Kessel's AES preprint #2729



$= 2.37$ Pa (peak)

$\Rightarrow$ 98.5 dB SPL

At 1kHz $\Rightarrow$ 138.5 dB

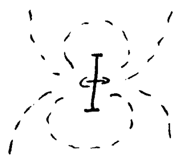
but.... breakup!

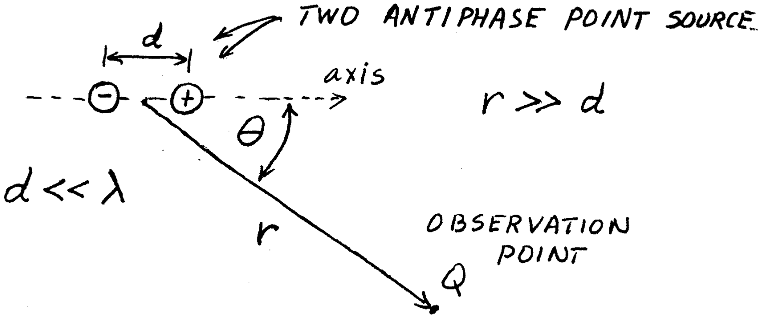
Tweeter 2.5 cm (1 inch), ± 0.1 mm oscillating at 2kHz, 1 m away

$$p = \frac{(1.2)\,\pi\,(0.0125)^2\, 4\pi^2 (4) 10^6\, 10^{-4}}{2\pi} = 1.48 \text{ Pa}_{\text{peak}}$$

$\Rightarrow$ 94.4 dB SPL

FREE PISTONS, DIPOLES





$r \gg d$

$d \ll \lambda$

at Q:

$$p = \frac{\rho_0}{4\pi}\left[\frac{A\left(t-\frac{r}{c}+\frac{d}{2c}\cos\theta\right)}{r-\frac{d}{2}\cos\theta} - \frac{A\left(t-\frac{r}{c}-\frac{d}{2c}\cos\theta\right)}{r+\frac{d}{2}\cos\theta}\right]$$

"VOLUME JERK" $\left[\frac{m^3}{sec^3}\right]$

$$= \frac{\rho_0}{4\pi}\left[\underbrace{\frac{d}{rc}\frac{dA}{dt}}_{\text{far-field}} + \underbrace{\frac{d}{r^2}A}_{\text{near-field}}\right]\cos\theta$$

(larger than by kr)

Note – in far field, $p \propto$ Jerk

– overall $\cos\theta$ pattern

<u>Example</u> 20 cm (8inch) disk, free in air, at 1 m, oscillating ± 1 mm at 100 Hz, on axis.

Estimate $d \sim 10$cm, $\frac{dA}{dt} \to \pi a^2 \omega^3 x$

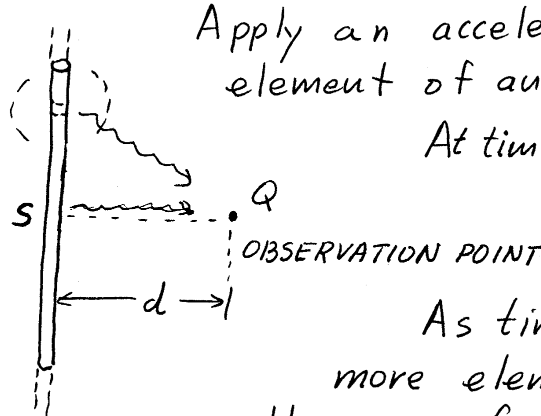
We find $p \sim 0{\cdot}22$ Pa peak

$\boxed{\text{BAFFLED RESULT WAS } 98{\cdot}5\text{ dB}}$ $\longrightarrow$ $\to 77{\cdot}8$ dB SPL

at 1kHz, $\Rightarrow$ 138 dB SPL $\left\{\begin{matrix}\text{cf}\\ \text{baffle}\end{matrix}\right\}$

Low sensitivity

LINE SOURCES



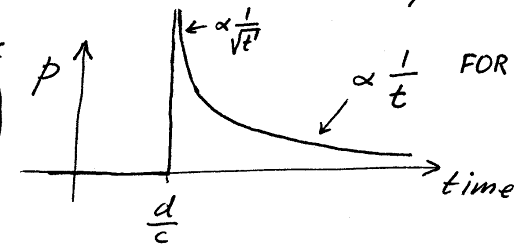
Apply an acceleration pulse to each element of an infinite line source.

At time $\frac{d}{c}$, first signal arrives from the region S.

As time passes, more and more elements contribute, but they are further away.

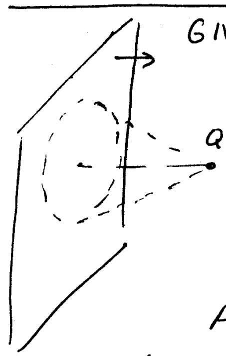
Net result:

see AES preprint #2417 for more information on line sources



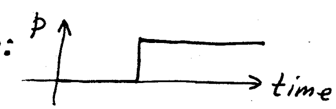
We can show this represents a 3 dB/oct pink spectrum, so the line needs equalization (with anti-pink filter).

PLANE WAVE SOURCE



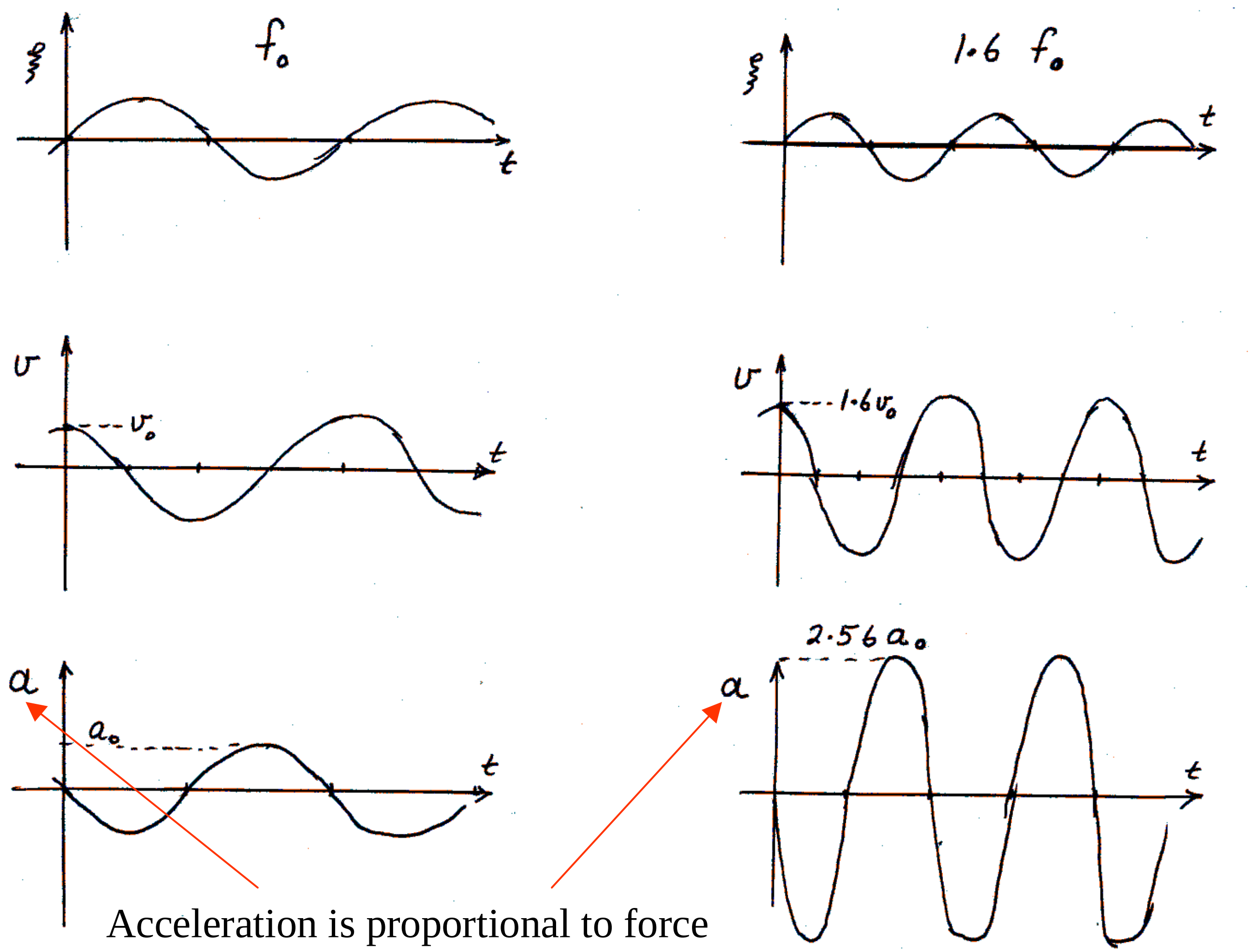
GIVE WALL AN ACCELERATION PULSE

AT Q WE FIND:



BUT THAT'S WHAT WE WOULD EXPECT, BECAUSE AN ACCELERATION PULSE LEAVES THE WALL MOVING, AND THE PRESSURE SHOULD BE GIVEN BY $p = \rho_0 c v$. (also needs eq. w.r.t. point source)

Kinematics and dynamics of shaking a mass



Conclusions: (1) Displacement is opposite direction to force !

(2) For constant displacement, acceleration is proportional to freq^2

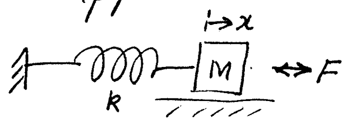
(3) For constant force, displacement is proportional to 1/freq^2

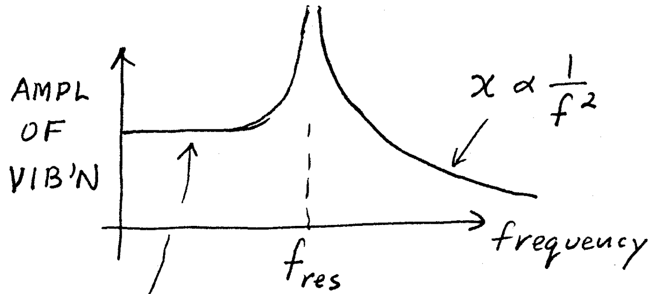
The 1925 Rice-Kellog loudspeaker model

DIRECT RADIATOR LOUDSPEAKERS

RESONANCE & ACCELERATION

Suppose we apply an oscillatory force F to a mass-spring system.

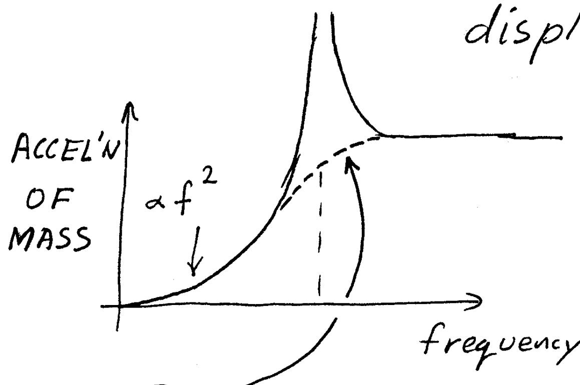




x independent of f.

Above resonance the amplitude of vibration $\propto \frac{1}{f^2}$

The acceleration of the mass is a factor f^2 relative to displacement amplitude.

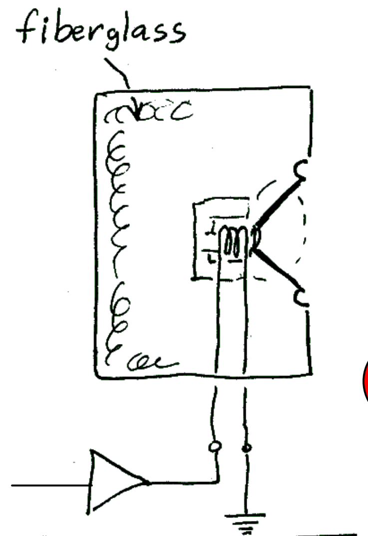


So above resonance, the acceleration is independent of frequency.

$\boxed{\text{Thus acoustic output is flat, for } ka < 1.5}$

Damping, such as due to the magnet and coil system of a **driver** unit fed from a low output impedance amplifier, can control the resonance peak.

IDEAL WOOFER RESPONSE



Spring is air in box, and marginally the woofer suspension.

Mass is the cone, and the air near it.

$$\text{Force} = B\,l\,i$$

B: magnetic flux density

l: total length of voice coil wire

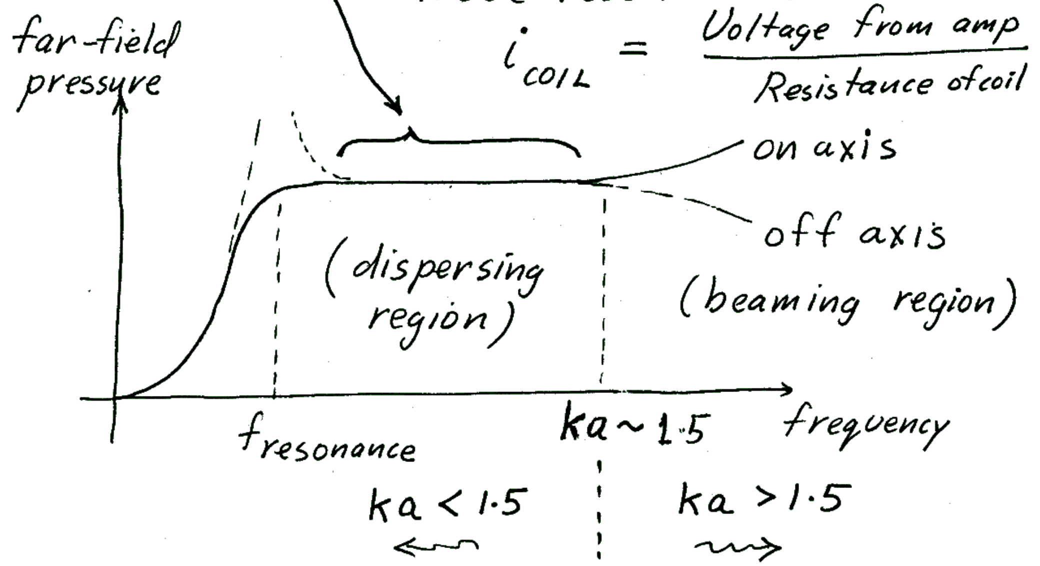
i: current in coil

Acoustic far-field output proportional to voltage on speaker coil

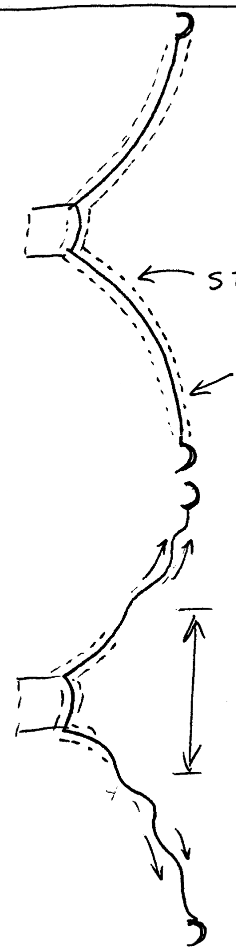
Only at resonance is the induced voltage significant.

Above resonance:

$$i_{COIL} = \frac{\text{Voltage from amp}}{\text{Resistance of coil}}$$



CONTROLLED BREAKUP



Cone motion at
low frequencies
(piston-like)

stiff here

less stiff here as flare
angle increases

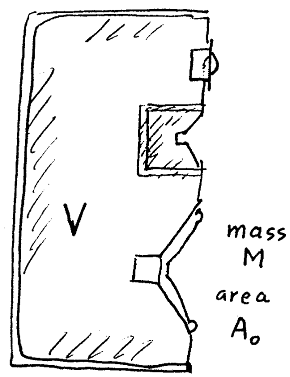
Cone motion at
higher frequencies

Damped ripples move
out from the centre
to the edges.

Effective radiating
area is reduced,
counteracting the
Huygen's principle tendency
to beaming forward.

Effective mass is reduced, to
keep up the output.

MIDRANGE, TWEETER



$\gamma = 1.4$ (air)

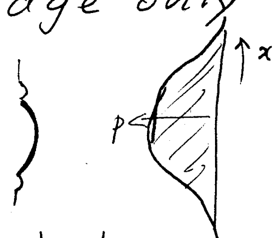
$p_0 \cong 10^5\ N/m^2$

$$f_{res} = \frac{1}{2\pi}\sqrt{\frac{\gamma\, p_0\, A_0^2}{M\ V}}$$

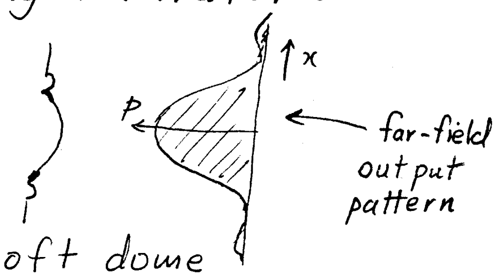
WOOFER BOX 25 ℓ	$f_{res} \sim$ 50 Hz
MIDRANGE $f_{res} \sim$ 400 Hz	Thus $V \lesssim 0.4\ \ell$
TWEETER $f_{res} \sim$ 1.5 kHz	A_0 much smaller, needs $\sim$ few cm^3

B&W philosophy is to: absorb the rear wave!

Soft domes: At high frequencies, a soft dome radiates at the coil edge only → ring radiator.



hard dome
single piston,
but at breakup,
watch out!

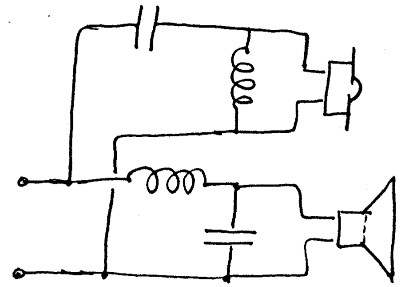


soft dome
somewhat narrower
pattern, more
side lobes (halo?)

CROSSOVER POINT : WHY

- NEED LARGE CONE AND DRIVER ASSEMBLY FOR ADEQUATE LOWS.
- BUT..., AT HIGHER FREQUENCIES...
  - → BEAMING ~ 1 kHz (POWER RESPONSE)
  - → PROBLEM GETTING HF TO CONE
  - → EVEN CONTROLLED BREAKUP HAS SHADOWED OUTPUT.
- FOR HIGHER BASS OUTPUT, CONE SIZES ARE LARGER, SO 3 WAY, BUT CROSSOVER IN 400 Hz RANGE IS CRITICAL, ACOUSTIC SUM FLAT.
- CROSSOVER CHOSEN TO
  - AVOID DRIVER IRREGULARITIES
  - CURTAIL POWER TO A UNIT

(keep high power out of tweeter, for example)



{TYPICAL 2nd ORDER MAY NEED TWEETER INVERTED POLARITY}

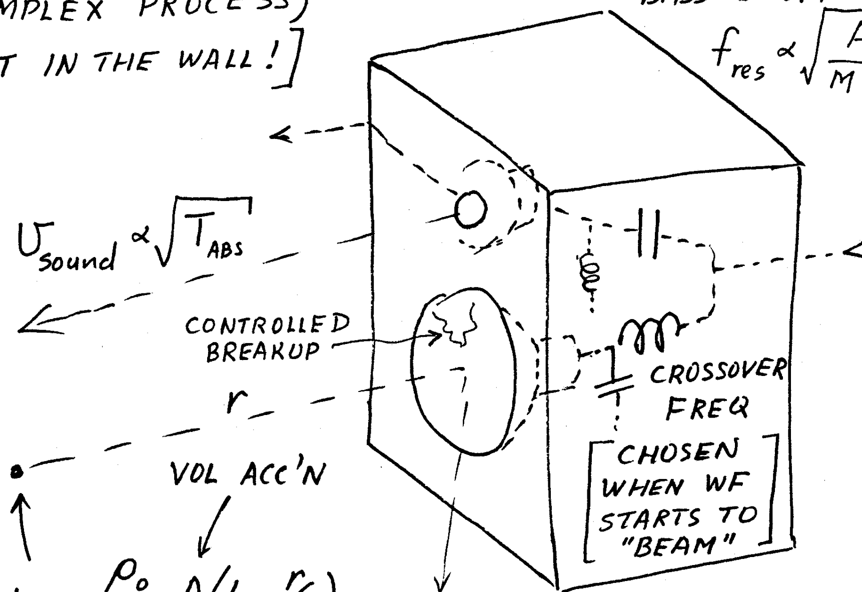
PUTTING IT ALL TOGETHER

(the main points)

EDGES OF BOX
DIFFRACT SOUND
(COMPLEX PROCESS)
[PUT IT IN THE WALL!]

BOX SIZE AND
CONE MASS AND AREA
DETERMINE LOW-FREQ
BASS CUTOFF

$$f_{res} \propto \sqrt{\frac{A_{cone}^2}{M\,V}}$$



$$p = \frac{\rho_0}{4\pi r} A(t - r/c)$$

ALL DRIVERS OPERATE
AS COMPACT SOURCES,
SO THEY DISPERSE WELL

WITH CONSTANT ACCELERATION DRIVE,
THEY ARE THEN FLAT, MEANING
CONSTANT COIL VOLTAGE AS
FN OF FREQUENCY

TRADE-OFFS: FOR A SMALL BOX, USE A HEAVY
CONE TO KEEP RESONANCE FREQ LOW.
THEN EFFICIENCY DROPS BUT RESPONSE NICE

The End